Optimal Fiscal Transfers in a Monetary Union*

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We derive the optimal fiscal transfer scheme for countries in a monetary union to offset the welfare losses resulting from asymmetric shocks and nominal rigidities. Optimal transfers involve a tradeoff between reducing national output gaps and the provision of consumption insurance across countries, where the weight of the former increases relative to the latter as consumption home bias rises. The welfare gains from optimal transfers increase in both home bias and export substitutability. When these parameters are calibrated to the data for specific euro area countries, the welfare gains from optimal transfers are as high as 3.6% of permanent consumption.

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1 Introduction

In a monetary union that faces nominal rigidities, asymmetric shocks across union members cannot be offset by the common central bank. Lowering the policy interest rate to stimulate member economies in recession runs the risk of overheating member economies at or above full employment output. Optimal policy requires the central bank to stabilize union wide average inflation, leaving a combination of fiscal policy and financial market integration to fulfill the idiosyncratic stabilization role that the central bank cannot play.[1] While federations like the United States, Canada, the U.K., and Germany smooth over 80 percent of the impact of local shocks through a combination of fiscal transfers and financial market integration, the euro area only insures half that amount with no centralized fiscal authority or fiscal union and comparatively little transnational risk-sharing through financial markets (IMF (2013)).

The relative paucity of cross-border risk-sharing in the euro area has prompted much debate about the need for a fiscal union between member states. However, there is still considerable uncertainty about the optimal design as well as the potential welfare gains resulting from such a union. Much of the academic research on fiscal unions has been conducted under the assumption of perfect cross-country risk-sharing, usually for tractability sake, where full consumption insurance is provided by internationally complete asset markets or by terms of trade movements.[2] The potential welfare gains from a fiscal union in these setups are very small, as the risk-sharing benefits of integration are nullified by assumption.

In contrast with the literature, we study the optimal design of a fiscal union in an open economy model with *incomplete cross-country risk-sharing*. We define a fiscal union as a revenue neutral stabilization program, facilitated by cross-country transfers, that insures member states against the vagaries of asymmetric shocks over the business cycle. We lay down a tractable model of a monetary union composed of a continuum of small open economies which face incomplete financial markets, nominal rigidities and asymmetric shocks and solve for the optimal fiscal union analytically, and then quantify the welfare gains from a fiscal union in a more general setup.

In the model there are two distortions that move allocations away from the Pareto optimum: a labor wedge which reflects deviations from the flexible wage allocation and is proportional to the output gap, and a consumption wedge which reflects deviations from the complete markets allocation.[3] The log-linearized labor wedge is zero in the flexible wage allocation, while the log-linearized consumption wedge is zero in the complete markets allocation. The labor wedge simply corresponds

[1]Beetsma and Jensen (2005), Gali and Monacelli (2008) and Ferrero (2009) show that monetary policy should stabilize inflation at the union level and that cooperative fiscal policy in the form of government spending has a country-specific stabilization role.

[2]See for example Beetsma and Jensen (2005), Gali and Monacelli (2008), Ferrero (2009) and Farhi and Werning (2017).

[3]We abstract from terms of trade externalities, which arise from a country's incentive to exploit monopoly power over its export products, as these are not a primary concern for designing a fiscal union. Throughout the paper we focus on cooperative allocations with steady state labor subsidies that remove the distortive impact of monopolistic steady state markups on exports.

to the distance between the marginal cost and benefit of labor in each country. Whenever the product of the real wage and the marginal utility of consumption exceeds the marginal disutility of labor, the labor wedge is positive. The consumption wedge corresponds to the efficiency of the transfer allocation. The marginal utility of the transfer is equal to the marginal utility of consumption divided by the domestic consumer price index. In complete markets the marginal utility of an additional transfer unit is equal across countries. If the marginal utility of an additional transfer unit is larger than its average value in the union, the consumption wedge is positive.

We prove that the optimal transfer scheme stabilizes a weighted average of the labor and consumption wedges, where the former enters with a weight equal to consumption home bias and the latter one minus home bias (*i.e.* openness). As home bias increases, more income is spent on domestically produced goods, such that transfers provide a larger boost to labor demand and are more influential in closing the labor wedge. Minimizing the labor wedge is thus more important for an economy with a high proportion of domestic goods in the consumption basket. On the other hand, as home bias shrinks, transfers have less impact on the labor wedge as more spending goes to imported goods. In more open economies, the optimal transfer places relatively more weight on minimizing the consumption wedge than the labor wedge. In general, the benefit of fiscal transfers increases with consumption home bias, as a larger proportion of transfer spending goes towards domestic goods, thereby raising labor demand and shrinking the domestic output gap. A fiscal union is thus more effective when home bias is strong.

In the limit, for a country with no home bias, we obtain a global closed-form solution to the model. Transfers do not affect labor demand or domestic production in a fully open economy, as all income is spent on imports. Although the labor wedge may generate large welfare losses, particularly when domestic products are highly substitutable with foreign products, transfers are powerless to ameliorate this distortion when domestic households consume a basket of goods made up entirely of imports. The optimal transfer scheme in a fully open economy thus focuses exclusively on minimizing the consumption wedge and replicates the decentralized complete markets allocation.

Different from Farhi and Werning (2017), we solve for optimal transfers without restrictive assumptions on household risk aversion or the substitutability of goods across countries — so-called trade elasticities. Farhi and Werning employ the widely used Cole-Obstfeld (1991) calibration which specifies log utility in consumption as well as unitary trade elasticities. Under this calibration full consumption risk-sharing is provided by terms of trade movements, as export revenues are constant in the presence of asymmetric productivity shocks due to offsetting income and substitution effects. While the Cole-Obstfeld/Farhi-Werning calibration is attractive for its analytical tractability, it significantly dampens the adverse impact of asymmetric productivity shocks on the labor and consumption wedges.

As domestic goods become closer substitutes with foreign goods, a negative home productivity shock leads to a larger decline in home exports in the absence of currency or wage adjustment, due to increased expenditure switching from home consumers. Larger export losses cause a greater fall in labor demand and aggregate demand, which further diminishes labor and consumption and raises their respective wedges. The Cole-Obstfeld/Farhi-Werning calibration mitigates this negative feedback loop and the distortionary impact of higher product substitutability. Recent estimates of aggregate trade elasticities for euro area countries fall in the range of three to five (Corbo and Osbat (2013), Imbs and Méjean (2015)). This has important implications for welfare, as higher export substitutability (higher trade elasticity) increases the consumption and labor wedges, which raises the cost of business cycle fluctuations in a monetary union, and strengthens the need for a fiscal union to offset the negative impacts of asymmetric shocks across countries.

To quantify the potential welfare gains from a fiscal union, we calibrate consumption home bias and trade elasticities to recent empirical estimates for euro area countries in an extended version of the model that includes imperfect cross-country risk-sharing, asymmetric shocks and Calvo wage rigidity.[4] We then solve for the optimal fiscal union numerically and demonstrate that the welfare gains from optimal transfers are potentially much larger than previous studies, including Farhi and Werning (2017), have documented. We confirm that transfers are more effective as home bias increases and product substitutability rises. Indeed, the welfare gains from a transfer union under the Cole-Obstfeld/Farhi-Werning calibration are extremely small, on the order of 0.01% of permanent consumption, relative to the gains using country-specific elasticity estimates from the data, which range from one to three percent of permanent consumption under incomplete markets. We also show that business cycle fluctuations in a monetary union are more costly by up to two orders of magnitude relative to an identical closed or flexible exchange rate economy. In contrast, welfare losses are low regardless of openness or substitutability for countries outside a monetary union, as exchange rate depreciation acts as an automatic stabilizer and significantly dampens any drop in domestic production.

Evers (2012) takes a more quantitative approach and estimates the welfare gains from a variety of transfer rules within a monetary union. He uses numerical solution methods and does not solve for the optimal transfer scheme explicitly. The distortions in his paper are fundamentally similar to our setup: deviations from the output gap and underinsurance as well as wage and price dispersion, both of which comove with the output gap. He also includes physical capital in his model, however relative to our framework this does not add any new distortions nor alter the intuition for the role of home bias or trade elasticities in the optimal transfer scheme. Evers (2012) finds that stabilizing labor income is the optimal policy even though it generates high consumption and GDP volatility. This is consistent with our results, as economies are relatively closed (home bias is 70%) in his calibration, and we prove that optimal transfers focus on the labor wedge more than the consumption wedge as home bias increases. In a similar setup to his 2012 paper, Evers (2015) studies the effectiveness

[4] Calvo wages introduce wage dispersion in addition to the consumption and labor wedges we discussed earlier. However, wage dispersion is proportional to the labor wedge.

of various forms of fiscal federalism in a monetary union and, consistent with our findings, shows that relative to the decentralized allocation with incomplete financial markets, a centralized transfer union decreases the volatility of consumption, labor and output amongst member economies, yielding positive welfare gains.

2 The Model With Wages Set One-Period-In-Advance

We consider a continuum of small open economies represented by the unit interval, as popularized in the literature by Gali and Monacelli (2005, 2008). Our model is based on Dmitriev and Hoddenbagh (2013), although here we consider wages rather than prices set one-period-in-advance, introduce home bias, and focus on a monetary union instead of flexible exchange rates.

Each economy consists of a representative household and a representative firm. All countries are identical ex-ante: they have the same preferences, technology, and wage-setting. Ex-post, economies will differ depending on the realization of their technology shock. Households are immobile across countries, however goods can move freely across borders. Perfectly competitive firms in each economy produce a country-specific variety, which can be either consumed at home or exported as an input in the foreign consumption basket. World demand for a country's unique export good is negatively sloped with a constant price elasticity, so that each economy exercises a degree of monopoly power over its variety. As in Corsetti and Pesenti (2001, 2005) and Obstfeld and Rogoff (2000, 2002), we use one-period-in-advance wage setting to introduce nominal rigidities. Workers set next period's nominal wages, in terms of domestic currency, prior to next-period's production and consumption decisions. Given this preset wage, workers supply as much labor as demanded by firms. We lay out a general framework below, and then hone in on the specific case of complete markets and financial autarky. To avoid additional notation, we ignore time subindices unless absolutely necessary. When time subindices are absent, we are implicitly referring to period t.

Production Each economy i produces a final good, which requires technology, Z_i, and aggregated labor, N_i. We assume that technology shocks are independent across time and across countries. We need not impose any particular distributional requirement on technology at this point. The production function of each economy will be:

$$Y_i = Z_i N_i. \tag{1}$$

Households, indexed by h, each have monopoly power over their differentiated labor input, which will lead to a markup on wages. A perfectly competitive, representative final goods producer aggregates differentiated labor inputs from households in CES fashion into a final good for export. Production of the representative final goods firm in a specific country is:

$$N_i = \left(\int_0^1 N_i(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is the elasticity of substitution between different types of labor, and $\mu_\varepsilon = \frac{\varepsilon}{\varepsilon-1}$ is the markup on labor.

The aggregate labor cost index, W, defined as the minimum cost to produce one unit of output, will be a function of the nominal wage for household h, $W(h)$: $W_i = \left(\int_0^1 W_i(h)^{1-\varepsilon} dh\right)^{\frac{1}{1-\varepsilon}}$. Cost minimization by the firm leads to demand for labor from household h:

$$N_i(h) = \left(\frac{W_i(h)}{W_i}\right)^{-\varepsilon} N_i. \tag{2}$$

In the open economy, monopoly power is exercised at both the household and the country level: at the household level because of differentiated labor, and at the country level because each economy produces a unique good. We show in Section 3 that optimizing cooperative policymakers will remove the household markup on labor through the distribution of output subsidies. In addition, we assume policymakers do not exploit the monopoly power over their country's unique export variety, which eliminates terms of trade externalities. Note that firms have no monopoly power and are perfectly competitive in the model.

Households In each economy, there is a household, h, with lifetime expected utility

$$\mathbb{E}_{t-1}\left\{\sum_{k=0}^{\infty} \beta^k \left(\frac{C_{it+k}(h)^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it+k}(h)^{1+\varphi}}{1+\varphi}\right)\right\} \tag{3}$$

where $\beta < 1$ is the household discount factor, $C(h)$ is the consumption basket or index, $N(h)$ is household labor effort (think of this as hours worked), σ is the risk aversion parameter, and φ is the inverse elasticity of labor supply. Households face a general budget constraint that nests both complete markets and financial autarky; we will discuss the differences between the two in subsequent sections. For now, it is sufficient to simply write out the most general form of the budget constraint:

$$C_i(h) = (1-\tau_i)\left(\frac{W_i(h)}{P_i(h)}\right) N_i(h) + \frac{\mathcal{D}_i(h)}{P_i} + \frac{\mathcal{T}_i(h)}{P_i} + \Gamma_i(h). \tag{4}$$

The distortionary tax rate on household labor income in country i is denoted by τ_i, while Γ_i is a domestic lump-sum tax rebate to households. $\mathcal{T}_i$ refers to lump-sum cross-country transfers. In the absence of a fiscal union, these cross-country transfers will equal zero ($\mathcal{T}_i = 0 \;\; \forall i$). Net taxes equal zero in the model, as any amount of government revenue is rebated lump-sum to households. The consumer price index corresponds to P_i, while the nominal wage is W_i. $\mathcal{D}_i$ denotes state-contingent portfolio payments expressed in real consumption units.

When international asset markets are complete, households perform all cross-border trades in contingent claims in period 0, insuring against all possible states in all future periods. The transverality condition simply states that all period 0 transactions must be balanced: payment for claims issued must equal payment for claims received. The transversality condition for complete markets is:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{-\sigma}}{P_{it}} \mathcal{D}_{it} \right\} = 0, \tag{5}$$

while in financial autarky $\mathcal{D}_{it} = 0$. Intuitively, the transversality condition stipulates that the present discounted value of future earnings should be equal to the present discounted value of future consumption flows. Under complete markets, consumers choose a state contingent plan for consumption, labor supply and portfolio holdings in period 0.

Consumption and Price Indices In each country i, the consumption basket consists of home $C_{H,i}$ and foreign $C_{F,i}$ goods,

$$C_i = \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,i})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,i})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{6}$$

where $C_{F,i}$ is defined as

$$C_{F,i} = \left(\int_0^1 (C_{F,ij})^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}}. \tag{7}$$

$C_{H,i}$ denotes consumption by households in country i of the domestically produced variety, while $C_{F,ij}$ denotes consumption by households in country i of the variety produced by country j. The elasticity of substitution between home and foreign products is defined by η, while the elasticity of substitution between the goods of different countries is defined by γ. The relative weight of home and foreign goods in the consumption basket is defined by the degree of home bias, $1-\alpha$. When $\alpha = 0$, households only consume domestic goods, while when $\alpha = 1$, the economy is fully open and households consume a basket made up entirely of imports.

The domestic price index consists of the prices of both home and foreign products:

$$P_i = \left[(1-\alpha)(P_{H,i})^{1-\eta} + \alpha (P_{F,i})^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{8}$$

The relative demand for home and foreign products is given by

$$C_{H,i} = (1-\alpha) \left(\frac{P_{H,i}}{P_i} \right)^{-\eta} C_i, \tag{9}$$

$$C_{F,i} = \alpha \left(\frac{P_{F,i}}{P_i} \right)^{-\eta} C_i. \tag{10}$$

The demand for country $i's$ production is

$$Y_i = (1-\alpha)\left(\frac{P_{H,i}}{P_i}\right)^{-\eta} C_i + \alpha\Big(P_{H,i}/P_{F,i}\Big)^{-\gamma} \int_0^1 C_{F,j}dj, \tag{11}$$

while goods market clearing for country i's unique variety is given by

$$Y_i = C_{H,i} + \int_0^1 C_{F,ji}dj. \tag{12}$$

Labor Market Clearing Households maximize lifetime utility (3) subject to the budget constraint (4). The first order condition for labor gives the labor supply condition (which is the optimal preset wage):

$$W_{it} = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{\frac{C_{it}^{-\sigma}N_{it}}{P_{it}}\right\}}.$$

The representative firm in country i maximizes profit by choosing the appropriate amount of aggregate labor, leading to the familiar labor demand condition equating the real wage at time t (W_{it}/p_{it}) with the marginal product of labor (Z_{it}). To obtain the labor market clearing condition, set labor demand equal to labor supply, and use the fact that the wage is preset at time $t-1$:

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}Y_{it}\frac{P_{H,it}}{P_{it}}\right\}}. \tag{13}$$

Taking the expectations operator out of (13) yields the flexible wage equilibrium.

Since shocks are independent across time and across countries, nominal preset wages are identical across countries so we drop the country-specific subindex i for wages:

$$W = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{\frac{C_{it}^{-\sigma}N_{it}}{P_{it}}\right\}}. \tag{14}$$

Domestic prices are given by:

$$P_{H,i} = \frac{W}{Z_i}. \tag{15}$$

We normalize the price of the foreign consumption basket to one, again ignoring the country-specific subindex i as P_F is identical across countries:

$$P_F = \left(\int_0^1 P_{H,i}^{\frac{\gamma-1}{\gamma}}\, di\right)^{\frac{\gamma}{\gamma-1}} = 1. \tag{16}$$

Given (15) and (16), we can express the nominal wage as a function of productivity in country i:

$$W = \left(\int_0^1 Z_i^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}. \tag{17}$$

Complete Markets

In complete markets, agents in each economy have access to a full set of domestic and foreign state-contingent assets. Households in all countries will maximize (3), choosing consumption, leisure, and a complete set of state-contingent nominal bonds, subject to (4). Complete markets imply the following risk-sharing condition:

$$\frac{\frac{C_{it}^{-\sigma}}{P_{it}}}{\frac{C_{it+1}^{-\sigma}}{P_{it+1}}} = \frac{\frac{C_{jt}^{-\sigma}}{P_{jt}}}{\frac{C_{jt+1}^{-\sigma}}{P_{jt+1}}} \quad \forall i, j \tag{18}$$

which states that the ratio of the marginal utility of consumption at time t and $t+1$ must be equal across all countries. This condition does not imply that consumption is equal across countries. Consumption in country i will depend on the initial asset position, fiscal policy, the distribution of country-specific shocks, the covariance of global and local shocks, and other factors.[5]

When (5) and (18) hold, we obtain the following expression for consumption in country i:

$$C_{it} = \frac{\mathbb{E}_0 \left\{ \sum_{t=1}^{\infty} \beta^t Y_{it} P_{H,it} \right\}}{\mathbb{E}_0 \left\{ \sum_{t=1}^{\infty} \beta^t P_{it}^{\frac{\sigma-1}{\sigma}} \right\}} P_{it}^{-\frac{1}{\sigma}} \tag{19}$$

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Financial Autarky

The aggregate resource constraint under financial autarky specifies that the nominal value of output in the home country must equal the nominal value of consumption in the home country:

$$C_i = Y_i \frac{P_{H,i}}{P_i} + \frac{\mathcal{T}_i}{P_i}. \tag{20}$$

Note that the sum of all transfers across countries in a given period must equal zero ($\int \mathcal{T}_i di = 0$), as the transfer scheme is revenue neutral in the aggregate. In the absence of a transfer union, there will be no transfers ($\mathcal{T}_i = 0 \; \forall i$).

[5]A policy change in economy i may lead to a change in consumption. For example, if the government taxes consumption, households will consume less in the long-run relative to the rest of the world. In spite of this, it is still possible to characterize an optimal consumption plan that is robust to changes in monetary and fiscal policy.

3 Solving the Model

Ultimately, our goal is to study different institutional arrangements and evaluate the size of the welfare costs of business cycles in each system in an effort to understand which distortions are most critical and how they can be ameliorated via policy arrangements, either at the subnational, national or international level.

We focus our attention here on two allocations. First, we examine the case of a monetary union where countries have flexible wages and access to complete markets, which coincides with the Pareto optimum. Second, we deviate from the Pareto optimum and consider a currency union with incomplete cross-country risk-sharing, sticky wages and access to state-contingent transfers from a fiscal union. In all allocations we assign cooperative steady-state production subsidies to offset monopolistic markups, thereby nullifying the negative welfare effect of terms of trade externalities.

Within a monetary union, a single central bank sets monetary policy for the union as a whole. In the presence of aggregate shocks, the union-wide central bank will stabilize inflation at the union level, a result shown in Gali and Monacelli (2008). However, for tractability we assume no aggregate shocks, only asymmetric country-specific shocks. With only one policy instrument, the union-wide central bank cannot eliminate wage rigidity at the country level in the presence of asymmetric shocks. As a result, the union-wide central bank does nothing in our model. None of our results change if we add aggregate shocks: these shocks would simply be counteracted by the union-wide central bank.

The Pareto Optimal Allocation

Proposition 1 ***The Decentralized Flexible Wage, Complete Markets Allocation*** *The household will maximize lifetime utility (3) subject to the budget constraint (4) and the transversality condition (5). The decentralized allocation under flexible wages, access to complete markets and cooperative steady-state subsidies is given by*

$$\frac{\chi N_i^{\varphi}}{Z_i} = \frac{C_i^{-\sigma} P_{H,i}}{P_i}, \tag{21a}$$

$$\frac{C_i^{-\sigma}}{P_i} = constant, \tag{21b}$$

as well as the domestic price index (8) and demand for the domestically produced variety (11).

Proof See Appendix A.1. ■

Corollary 1. ***The Pareto Optimum*** *The social planner will maximize the equally weighted joint utility of all i countries in the monetary union,*

$$\int_0^1 \left[\frac{C_i^{1-\sigma}}{1-\sigma} - \chi \frac{N_i^{1+\varphi}}{1+\varphi} \right] di \tag{22}$$

subject to the production function (1), the aggregate domestic consumption basket (6), the import basket (7), and goods market clearing (12). The solution to this maximization problem yields the Pareto optimum, which corresponds to the decentralized flexible wage, complete markets allocation with cooperative steady-state subsidies offsetting monopolistic markups, given by (21a) and (21b).

Proof See Appendix A.2. ■

Under the decentralized flexible wage allocation households set their wages to equate the marginal benefit of working an additional hour with the marginal cost. From (21a), we see that the marginal benefit is equal to the product of the real wage and the marginal utility of consumption, while the marginal cost corresponds to the disutility of labor. Under complete markets, households try to equalize the marginal benefit of contingent payments across states of the world. In doing so, households aim to achieve two goals. First, they want to stabilize their consumption in a manner proportional to their degree of risk aversion. Second, they prefer larger contingent claims in the union currency whenever the domestic price index is relatively small so that they have more units of consumption per unit of insurance.

Since the decentralized equilibrium with flexible wages and cooperative subsidies optimally allocates labor in each period, and state-contingent securities provide full consumption insurance against idiosyncratic productivity shocks, the flexible wage allocation with internationally complete asset markets is Pareto optimal.

The Monetary Union Allocation With Sticky Wages And Incomplete Markets

We now focus on allocations that deviate from the Pareto optimal allocation. In order to measure the distance of any allocation from the Pareto optimal allocation, we introduce the labor wedge and the consumption wedge below.

Definition of Labor and Consumption Wedges *Deviations from the Pareto optimal allocation can be measured in the residual for the labor optimality condition, what we call the labor wedge $V_{N,i}$, and the residual for the consumption optimality condition, what we call the consumption wedge $V_{C,i}$. Formally, we define the wedges as follows:*

$$V_{N,i} = \frac{C_i^{-\sigma} \frac{Z_i P_{H,i}}{P_i}}{\chi N_i^{\varphi}} \tag{23}$$

$$V_{C,i} = \frac{\frac{C_i^{-\sigma}}{P_i}}{\mathbb{E}\{\frac{C_i^{-\sigma}}{P_i}\}} \tag{24}$$

$$\hat{V}_{N,i} = -\varphi \hat{N}_i + \hat{Z}_i - \sigma \hat{C}_i + \hat{P}_{H,i} - \hat{P}_i \tag{25}$$

$$\hat{V}_{C,i} = -\sigma \hat{C}_i - \hat{P}_i + \sigma \mathbb{E}\{\hat{C}_i\} + \mathbb{E}\{\hat{P}_i\} \tag{26}$$

where $\hat{V}_{N,i}$ and $\hat{V}_{C,i}$ refer to the log-linear labor and consumption wedges respectively.

The log-linearized labor wedge is zero in flexible wage allocations, while the log-linearized consumption wedge is zero in complete markets allocations. Under the Pareto optimal allocation, both the log-linearized labor and consumption wedges equal zero ($\hat{V}_{N,i} = 0, \hat{V}_{C,i} = 0$). The labor wedge simply corresponds to the distance between the marginal cost and benefit of labor in each country. Whenever the product of the real wage and the marginal utility of consumption exceeds the marginal disutility of labor, the labor wedge is positive. Note that the labor wedge is proportional to the domestic output gap. The consumption wedge corresponds to the efficiency of the transfer allocation. Under the optimal allocation, transfers should go to the country where one unit of transfer generates the highest increase in consumption (which will correspond to those countries with a low domestic price index in the monetary union) and thus delivers the highest marginal utility.

As we move into an examination of the optimal transfer scheme, we will call on the labor and consumption wedges to provide intuition for our results. Below, we study allocations in a monetary union with sticky wages, incomplete cross-country risk-sharing and cross-country fiscal transfers. The consumption wedge will be non-zero in these allocations and will be affected by transfers.

Proposition 2 ***The Decentralized Sticky Wage, Incomplete Markets Allocation*** *The equilibrium for a member of a monetary union with sticky wages, no access to international financial markets, and access to cross-country transfers and steady state cooperative subsidies is described by domestic production (1), the home price index (8), labor market clearing (13), the price of the home good (15), the price index for foreign goods (16), the domestic real wage (17), and domestic consumption (20). The allocation is given by*

$$C_i = P_{H,i}^{1-\gamma} P_i^{-\eta} \mathbb{E}\{C_i P_i^{\eta}\} + \frac{1}{\alpha} P_i^{-\eta} \mathcal{T}_i, \tag{27a}$$

$$N_i = P_{H,i}^{-\gamma} P_i^{1-\eta} Z_i^{-1} \mathbb{E}\{C_i P_i^{\eta}\} + \frac{1-\alpha}{\alpha} P_{H,i}^{-\eta} Z_i^{-1} \mathcal{T}_i. \tag{27b}$$

To the first-order approximation, the above allocation can be expressed as:

$$\hat{C}_i = (\gamma - 1 + \eta(1-\alpha))\hat{Z}_i + \frac{1}{\alpha C_{ss}} \hat{\mathcal{T}}_i, \tag{28a}$$

$$\hat{N}_i = (\gamma - 1 + (\eta - 1)(1-\alpha))\hat{Z}_i + \frac{1-\alpha}{\alpha C_{ss}} \hat{\mathcal{T}}_i. \tag{28b}$$

where C_{ss} is steady state consumption.

Proof See Appendix A.3. ■

Under a transfer union, the volatility of consumption and labor increases in both trade elasticities (η, γ) as long as products are substitutes rather than complements (*i.e.* as long as η and γ are greater than one).[6] As the domestic consumption basket becomes more heavily weighted to imported products (*i.e.* as $\alpha \to 1$), consumption and labor become less sensitive to transfer income. In a fully open

[6]When home and foreign products are complements ($\eta < 1$), it is possible that hours worked will actually increase in response to a negative idiosyncratic productivity shock. This would imply that, for example, countries on the

economy, labor is completely insensitive to cross-country transfers. If goods are substitutes, openness also decreases the volatility of consumption and labor with respect to asymmetric productivity shocks.

In a monetary union with sticky wages, a fall in domestic productivity leads to a jump in the price of domestically produced goods. The resulting decline in demand for domestic production is stronger when home goods are more substitutable. Lower demand decreases consumption and hours worked. The volatility of consumption and labor thus increases with product substitutability. Consumption and hours worked are more insulated from asymmetric productivity shocks in economies with a higher proportion of imports in the consumption basket due to a smaller multiplier effect. A decline in exports leads to lower domestic aggregate demand, but in a very open economy domestic demand does not impact domestic production very much as most domestic income is spent on imports. Relative to economies with high consumption home bias, very open economies are heavily dependent on export revenues, but are less negatively impacted by a collapse in domestic demand. For the same reason, increasing openness diminishes the ability of transfers to mitigate the labor wedge, as a larger portion of income is spent on imports rather than domestic goods.

Optimal Transfers

The optimal fiscal union enhances the welfare of member economies by minimizing the consumption and labor wedges. However, since there is only one instrument for two wedges, the optimal transfer scheme stabilizes a weighted average of the two wedges.

Proposition 3 ***Optimal Transfers*** *Under the optimal transfer scheme, the policymaker maximizes representative household utility (3) subject to the consumption allocation from (27a), the labor allocation from (27b) and the optimal wage-setting condition (14). The requirement that all transfers add up to zero is already implied in (27a) and (27b). The optimal transfer allocation for a member of a currency union with sticky wages and incomplete markets is described by:*

$$C_i^{-\sigma} = (1-\alpha)\chi N_i^{\varphi} P_{H,t}^{-\eta} Z_i^{-1} P_i^{\eta} + \alpha\chi \mathbb{E}\{N_i^{\varphi} P_{H,t}^{-\gamma} Z_i^{-1}\} P_i^{\eta}, \tag{29a}$$

$$N_i = (1-\alpha) P_{H,t}^{-\eta} Z_i^{-1} P_i^{\eta} C_i + \alpha P_{H,t}^{-\gamma} Z_i^{-1} \mathbb{E}\{C_i P_i^{\eta}\}. \tag{29b}$$

To a first-order approximation, the optimal transfer scheme sets a weighted average of the labor and consumption wedges equal to zero:

$$(1-\alpha)\hat{V}_{N,i} + \alpha \hat{V}_{C,i} = 0. \tag{30}$$

Proof See Appendix A.4. ■

euro area periphery should have increased labor demand following negative productivity shocks. In the light of high empirical estimates for trade elasticities and low labor demand for euro area periphery countries, we focus mainly on calibrations where goods are substitutes. However, the analytical results hold whether home and foreign products are substitutes or complements.

The optimal transfer strategy for the union is to minimize the consumption wedge when the economy is relatively open, and the labor wedge when the economy is relatively closed. This is intuitive since transfers have a larger impact on labor demand in a relatively closed economy, as additional spending is directed toward domestically produced goods, while in more open economies most of the benefits of higher aggregate demand are reflected in higher imports. In an economy with no home bias, the optimal transfer union replicates the decentralized complete markets allocation, such that the log-linear consumption wedge will be zero. In such an economy, labor demand is completely independent of the size of fiscal transfers, as all goods produced in the economy are exported abroad, and the consumption basket is composed entirely of imported products.

4 Welfare Analysis In The Closed-Form Model With No Home Bias

We now analyze welfare across different allocations when economies have no consumption home bias. This special case provides sharp analytical insights since we obtain a global closed-form solution for each allocation. Rather than approximating a quadratic welfare function around a particular steady state, the advantage of the closed-form solution is that we can calculate welfare explicitly at any steady state. Under no home bias, the optimal transfer union and the decentralized complete markets allocations are identical since transfers in this scenario do not influence the labor wedge and thus policymakers focus only on minimizing the consumption wedge. The welfare analysis conducted here also provides intuition about the effect of product substitutability on wedges and welfare.

Our calibration for the closed-form model at quarterly frequency follows standard benchmarks from the literature and is reported in Table 1. In our welfare analysis, we allow the trade elasticities to vary while fixing the other parameters of the model.

Expected Utilities

Below we calculate the log of expected utility for four allocations: flexible wages with complete risk-sharing (31a) or financial autarky (31c); fixed wages (*i.e.* sticky wages) with complete markets (31b) or in financial autarky (31d).[7] Details on how we compute welfare analytically for each allocation are found in Appendix B. We assume technology is log-normally distributed in all countries, $\log(Z_i) \sim$

[7]Note that the flexible wage allocations also serve as a proxy for flexible exchange rates. In Appendix B we show that, for economies outside of a monetary union with a flexible exchange rate, the central bank will conduct policy to mimic the flexible wage allocation when faced with idiosyncratic productivity shocks.

$N(0, \sigma_Z^2)$, and is independent across time and across countries.

$$\log \mathbb{E}\left\{U_{\text{flex complete}}\right\} = \log u_{ss} + \frac{(\gamma-1)(1-\sigma)(1+\varphi)^2}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2 \tag{31a}$$

$$\log \mathbb{E}\left\{U_{\text{fixed complete}}\right\} = \log u_{ss} + \frac{(\gamma-1)(1-\sigma)(1+\varphi)(1+\varphi-\gamma\varphi)}{(\sigma+\varphi)}\sigma_Z^2 \tag{31b}$$

$$\log \mathbb{E}\left\{U_{\text{flex autarky}}\right\} = \log u_{ss} + \frac{(\gamma-1)(1-\sigma)(1+\varphi)^2\left[1+\varphi+(\gamma-1)(1-\sigma)(\sigma+\varphi)\right]}{(\sigma+\varphi)[1-\sigma+\gamma(\sigma+\varphi)]^2}\sigma_Z^2 \tag{31c}$$

$$\log \mathbb{E}\left\{U_{\text{fixed autarky}}\right\} = \log u_{ss} + \frac{(\gamma-1)(1-\sigma)(1+\varphi)\left[1-(\gamma-1)(\sigma+\varphi)\right]}{\sigma+\varphi}\sigma_Z^2 \tag{31d}$$

Using these expected utilities, it is straightforward to show that: (1) improved risk-sharing always has positive welfare consequences, (2) flexible wages always have positive welfare consequences, and (3) the gains from improved risk-sharing are always higher within a monetary union than outside of one.[8] We also see that under the Cole-Obstfeld (1991) calibration with log utility and unitary elasticity ($\sigma = \gamma = 1$), the expected utility for all policy coalitions is identical. In particular, unitary elasticity leads to complete risk-sharing via terms of trade movements which stabilizes both consumption and labor.

Figure 1 plots consumption in each allocation as a percentage of the Pareto optimum. The negative impact of the wage rigidity distortion dominates the negative impact of financial autarky, as both fixed wage allocations perform quite poorly relative to the flexible wage allocations, particularly as the degree of substitutability increases. The relative similarity of all flexible wage allocations (Flex Complete, Flex Autarky) is quite striking. Even in financial autarky, flexible wages enable households to stabilize consumption with small movements in their labor hours. The benefit of consumption risk-sharing is thus very small when wages are flexible. The gains from flexible wages approach 2% of permanent consumption under complete risk-sharing and 4% of permanent consumption under financial autarky for $\gamma = 10$. On the other hand, the welfare gains from perfect risk-sharing via a transfer union or complete markets equal 2% of permanent consumption within a monetary union when goods are highly substitutable ($\gamma = 10$).

5 Welfare Analysis In The Extended Model

In this section we quantify the potential welfare gains from a fiscal union in a model identical to the one outlined in Section 2, except we employ Calvo wage rigidity rather than one-period-in-advance wage ridigity and add a non-contingent safe government bond to the international financial market. The model presented here is laid out in detail in Appendix C. Calvo wage rigidity adds the distortive impact of wage dispersion to the decentralized allocations. Wage dispersion comoves with the labor

[8]See Appendix B equations (B.15a) – (B.15d) for calculation of explicit welfare differences.

wedge, and thus increases the importance of minimizing the labor wedge relative to the consumption wedge in the optimal transfer union.

We calibrate the extended model at quarterly frequency. All parameter values are found in Table 1. The elasticity of substitution between home and foreign products is defined by η, while the elasticity of substitution between the goods of different countries remains γ. The relative weight of these goods in the consumption basket is defined by the degree of home bias, $1 - \alpha$. When $\alpha = 0$, households only consume domestic goods, while when $\alpha = 1$, the economy is fully open and households will consume a basket made up entirely of imports from all other countries in the world. The strength of wage rigidity is defined by θ_W, the fraction of households who are able to reset wages in each period. We consider three primary settings for Calvo wage rigidity: flexible wages ($\theta_W = 0$), low wage rigidity ($\theta_W = 0.75$) which implies that the average household resets wages once every four quarters, and high wage rigidity $\theta_W = 0.87$ which implies that the average household resets wages every two years.[9]

We calculate the welfare losses from business cycle fluctuations in four different allocations: financial autarky, incomplete markets and complete markets in the absence of a fiscal union, and finally under the optimal transfer scheme. To conduct these welfare comparisons across different allocations, we follow Lucas (2003) and estimate the utility from a deterministic consumption path and a risky consumption path with the same mean. We calculate the amount of consumption necessary to make a risk-averse household indifferent between the deterministic and risky consumption streams. As we have done throughout the paper, in all allocations we remove terms of trade externalities through cooperative output subsidies so that there are no steady state markups on exports.

Figure 2 plots the loss in permanent consumption from business cycle fluctuations in financial autarky for high wage rigidity ($\theta_W = 0.87$). In this environment, home bias lowers welfare for every value of the trade elasticity for domestic exports γ. The losses in permanent consumption in financial autarky in a currency union are as high as 16% when economies are completely open ($\alpha = 1$) and 17% under full home bias ($\alpha \to 0$) when trade elasticity is very high ($\gamma = 10$). In a fully open economy, the decline in exports and income leads to a decrease in consumption and increases the consumption wedge, this consumption decline causes no further effect on income and labor wedge as the latter is derived from export. On the other hand, in the presence of home bias the consumption wedge and labor wedge become more related, as decline in consumption drives domestic production down and kickstarts a standard Keynesian multiplier effect between consumption and income. Thus, the volatility of business cycles grow with higher home bias.

[9] We take $\theta_W = 0.75$ as a conservative estimate of wage rigidity from Basu, Barattieri, and Gottschalk (2014), who find strong empirical evidence for θ_W in the range of 0.75 and 0.8 using U.S. micro data. Even $\theta_W = 0.87$ is a relatively conservative parameterization given recent estimates by Schmitt-Grohe and Uribe (2012). They find very strong downward wage rigidity in a number of countries in Europe from 2008-2011, including Greece, Portugal, and Spain. Although wages are more flexible in the upward direction, our focus here is on the negative effect of downward wage rigidity and the large welfare losses that accrue in a currency union under this scenario. Cacciatore, Ghironi and Fiori (2015) show that labor market reforms which reduce downward wage rigidity in a monetary union provide large welfare gains.

Figure 3 plots the loss in permanent consumption from business cycle fluctuations in incomplete markets. The ability to trade safe government bonds greatly improves welfare for countries in a currency union who are exposed to asymmetric shocks. Different from financial autarky, an increase in home bias improves the ability of countries to stabilize business cycles when markets are incomplete precisely because the consumption wedge and the labor wedge become more related. If a country is completely open, negative productivity shocks generate capital inflows which stabilize household income and consumption, thereby shrinking the consumption wedge. However, positive capital inflows will have no impact on hours worked or domestic output in a fully open economy, as all income is spent on imports. The labor wedge thus increases, as domestic production is sold exclusively abroad as exports, and exports become uncompetitive. Under incomplete markets, stronger home bias acts as a stabilizing force for consumption and hours worked, and shrinks both the consumption and labor wedges because firms supply more goods to the home market and thus an increase in domestic income and consumption through capital inflows also stabilizes labor and output.

The welfare losses arising from business cycle fluctuations under the optimal transfer structure are shown in Figure 4. Qualitatively, the economy in a fiscal union performs in a similar manner to the incomplete markets case. Optimal transfers deliver higher welfare gains for relatively closed economies with a higher elasticity of substitution. Transfers stabilize consumption, which lowers labor volatility particularly for countries that consume a high proportion of domestically produced goods. Quantitatively the optimal transfer union delivers higher welfare gains than incomplete markets. While households try to stabilize their consumption and maximize risk-sharing under incomplete markets, optimal transfers improve welfare by focusing on the tradeoff between the labor and consumption wedges. For example, in response to a negative shock transfers may increase consumption more than is necessary for the provision of full consumption insurance, in order to stabilize labor volatility. In other words, the social planner recognizes the demand externality arising from increased household consumption, which decreases wage rigidity and produces increased labor demand for other households. Overall, a fiscal union can successfully address asymmetric productivity shocks for sufficiently closed economies even when trade elasticities are high, since transfers are more effective in mitigating the labor wedge as economies become less open. However, very open economies with highly substitutable exports remain vulnerable to the threat of asymmetric productivity shocks in a monetary union. Their welfare losses can be large even under the optimal transfer structure, as their labor wedge is relatively insulated from the impact of transfer income.

As a robustness check, we plot the loss in permanent consumption from business cycle fluctuations in financial autarky for varying levels of Calvo wage rigidity in Figure 5. We set consumption home bias equal to the euro area average of 65% (*i.e.* $(1-\alpha) = 0.65$). Again, we see that it is not only wage rigidity that leads to large welfare losses, but the combination of wage rigidity *and* high trade elasticities, as both factors increase labor wedge. Simply put, when a country in a monetary union produces exports that are easily substitutable, the welfare consequences are dramatic. Coupled with rigid wages, as the evidence in Schmitt-Grohe and Uribe (2011) suggests for some European countries, the losses in permanent consumption are small for low subsitutability but quite large under high substitutability. Even for conservative estimates of wage rigidity, the losses in permanent consumption are considerable for high trade elasticities.

Overall, we show that as home bias increases, the consumption and labor wedges become more strongly related. Higher wage rigidity strengthens the labor wedge for all allocations, while higher trade elasticities strengthen both the consumption and labor wedges. Higher home bias under financial autarky has a destabilizing effect as growing labor and consumption wedges reinforce each other. On the other hand, under incomplete markets, complete markets and the optimal transfer union, home bias is welfare enhancing as a more stable consumption wedge through capital inflows also stabilizes domestic output and labor.

Welfare Losses for Country-Specific Elasticity Estimates

We compute the welfare losses from business cycles for a number of European countries under financial autarky, incomplete markets, complete markets and the optimal transfer union resulting from 1% technology shocks. We consider three different values for Calvo wage rigidity in our computations: high wage rigidity ($\theta_W = 0.87$), low wage rigidity ($\theta_W = 0.75$) and flexible wages ($\theta_W = 0$). Table 2 reports the loss in permanent consumption for the country-specific elasticity estimates of Corbo and Osbat (2013), while Table 3 reports the results for the country-specific elasticity estimates of Imbs and Méjean (2010). The results are particularly striking for countries in financial autarky with high trade elasticities, where the losses are as high as 7.22% (Greece). Access to a non-contingent bond for households significantly improves welfare, as permanent consumption losses drop to a range of 1.19% (Slovakia) to 2.88% (Austria) under high wage rigidity. Moving from incomplete markets to complete markets yields a smaller welfare gain of between 0.5 to 2 percent of permanent consumption for most countries in the sample. A transfer union is the most efficient allocation, as losses range from 0% (Italy) to 0.56% (Czech Republic). Consistent with our intuition from the baseline model, the welfare losses under the optimal transfer union increase with an economy's openness and the substitutability of its exports, as the labor wedge becomes less affected by transfers.

In Table 4 we compute the welfare losses in incomplete markets resulting from productivity shocks calibrated to match the volatility and autocorrelation of output in the data. The autocorrelation of HP-filtered output (ρ_Y) in the data is approximately 0.9998. We conservatively set $\rho_Z = 0.99$ for our simulations which implies $\rho_Y = 0.99$ in the model; if we increase ρ_Z, the welfare losses also increase. The volatility of productivity shocks ($\sigma_{Z,1}$, $\sigma_{Z,2}$) are calibrated to match the volatilities of HP-filtered output ($\sigma_{Y,1}$, $\sigma_{Y,2}$). We conduct welfare analysis for two scenarios. In Scenario 1 we take the volatility of output ($\sigma_{Y,1}$) from HP-filtered GDP data for each country with no adjustments and calibrate productivity shocks ($\sigma_{Z,1}$) to match the output volatilities from the data. In Scenario

2 we compute the volatility of output ($\sigma_{Y,2}$) from HP-filtered GDP data for each country after subtracting euro area GDP in order to construct a valid measure of asymmetric productivity shocks. We then calibrate the volatility of asymmetric productivity shocks ($\sigma_{Z,2}$) to match the volatility of asymmetric output in the data ($\sigma_{Y,2}$). The volatility of output resulting from asymmetric shocks ($\sigma_{Y,2}$) is roughly half of the overall volatility of output for each country ($\sigma_{Y,1}$). Scenario 1 may be thought of as a set of upper bound estimates and Scenario 2 as a set of lower bound estimates of empirically plausible welfare losses resulting from the absence of perfect cross-country risk sharing in a monetary union, where a common central bank cannot respond to asymmetric shocks across countries. The countries with the largest to gain from a transfer union are the smaller "periphery" countries, including Greece, Hungary, the Czech Republic, Portugal and Slovakia.

These results on the welfare gains of a transfer union over and above the complete markets allocation are consistent with the intuition we gleaned from the baseline model in Section (2), as well as with Farhi and Werning (2017), as we show that the privately optimal allocation under complete markets is inefficient and can be improved upon by government intervention in the form of fiscal transfers between countries. We then explicitly quantify the country-specific welfare gains from transfers without restricting ourselves to the Cole-Obstfeld calibration. Using country-specific calibrations, we document economically significant gains for a number euro area economies, particularly for those with little access to international financial markets and highly substitutable export products. Greece is a prime example of both.

6 Conclusion

In this paper we build a tractable model of a monetary union that faces nominal rigidities, imperfect risk-sharing across countries and asymmetric shocks and solve for the optimal fiscal union analytically, and then quantify the welfare gains from a fiscal union in a more general setup. In the model there are two distortions: a labor wedge which reflects deviations from the flexible wage allocation due to nominal rigidites and is proportional to the output gap, and a consumption wedge which reflects deviations from the complete markets allocation. We prove that the optimal transfer scheme stabilizes a weighted average of the labor and consumption wedges, where the former enters with a weight equal to consumption home bias and the latter one minus home bias (*i.e.* openness). As home bias increases, more income is spent on domestically produced goods, such that transfers provide a larger boost to labor demand and are more influential in closing the labor wedge. Minimizing the labor wedge is thus more important for an economy with a high proportion of domestic goods in the consumption basket. In general, the benefit of fiscal transfers increases with consumption home bias, as a larger proportion of transfer spending goes towards domestic goods, thereby raising labor demand and shrinking the domestic output gap. A fiscal union is thus more effective when home bias is strong.

To quantify the potential welfare gains from a fiscal union, we calibrate consumption home bias and trade elasticities to recent empirical estimates for euro area countries in an extended version of the model that includes imperfect cross-country risk-sharing, asymmetric shocks and Calvo wage rigidity. We then solve for the optimal fiscal union numerically and demonstrate that the welfare gains from optimal transfers are potentially much larger than previous studies, including Farhi and Werning, have documented. We confirm that transfers are more effective as home bias increases and as product substitutability rises. Indeed, the welfare gains from a transfer union under the Cole-Obstfeld/Farhi-Werning calibration are extremely small, on the order of 0.01% of permanent consumption, relative to the gains using country-specific elasticity estimates from the data, which range from one to three percent of permanent consumption under incomplete markets. We also show that business cycle fluctuations in a monetary union are more costly by up to two orders of magnitude relative to an identical closed or flexible exchange rate economy. In contrast, welfare losses are low regardless of openness or substitutability for countries outside a monetary union, as exchange rate depreciation acts as an automatic stabilizer and significantly dampens any drop in domestic production.

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Table 1: Calibration of the Closed-Form and Extended Models

Parameter	Value	Description
σ	2	Risk aversion parameter
φ	3	Inverse labor supply elasticity (Gali and Monacelli (2005, 2008))
χ	1	Following Gali and Monacelli (2005, 2008)
ε	6	Elasticity between different types of labor
θ_W	Varies	Calvo parameter for wage rigidity
β	0.99	Household discount factor
γ	Varies	Elasticity of substitution between home and foreign goods
α	Varies	Openness
η	1	Elasticity between home and foreign goods in extended model
ρ_Z	0.95	Persistence of technology shock in extended model
σ_Z	0.01	Standard deviation of technology

Table 2: Losses in Permanent Consumption for Country-Specific Elasticity Estimates of Corbo & Osbat (2013)

	Parameters			**High Wage Rigidity** ($\theta_W = 0.87$)				**Low Wage Rigidity** ($\theta_W = 0.75$)				**Flex Wages** ($\theta_W = 0$)	
	α	η	γ	Fin. Autarky	Incomp. Markets	Complete Markets	Transfer Union	Fin. Autarky	Incomp. Markets	Complete Markets	Transfer Union	Fin. Autarky	Incomp. Markets
Austria	0.55	4.5	3.8	6.05	2.88	2.09	0.37	2.76	1.41	1.08	0.33	0.032	0.011
Czech	0.70	3.4	3.8	5.13	2.86	2.21	0.56	2.40	1.41	1.13	0.47	0.029	0.010
Denmark	0.40	3.3	3.4	5.48	1.88	1.08	0.06	2.54	0.94	0.58	0.06	0.033	0.014
Finland	0.32	3.5	3.4	5.83	1.64	0.81	0.02	2.68	0.82	0.44	0.02	0.034	0.016
France	0.31	3.7	3.8	6.38	1.82	0.95	0.03	2.88	0.91	0.51	0.03	0.035	0.016
Germany	0.34	3.7	4.3	6.75	2.18	1.27	0.07	3.02	1.08	0.68	0.07	0.035	0.015
Greece	0.35	2.9	4.1	6.06	1.90	1.06	0.04	2.76	0.95	0.57	0.04	0.035	0.015
Hungary	0.49	3.3	4.2	6.06	2.62	1.80	0.23	2.76	1.29	0.94	0.21	0.033	0.013
Italy	0.22	3.2	3.2	5.67	1.01	0.30	0.00	2.61	0.51	0.17	0.00	0.036	0.019
Netherlands	0.62	3.5	3.5	5.07	2.55	1.85	0.34	2.38	1.26	0.96	0.30	0.029	0.010
Portugal	0.41	3.3	3.9	5.97	2.18	1.34	0.10	2.73	1.08	0.71	0.09	0.034	0.014
Slovakia	0.30	3.7	3.9	6.50	1.81	0.93	0.03	2.92	0.90	0.50	0.02	0.035	0.016
Spain	0.30	3.4	3.2	5.61	1.44	0.65	0.01	2.59	0.72	0.35	0.01	0.035	0.017
Sweden	0.42	4.2	4.5	6.97	2.77	1.87	0.21	3.09	1.36	0.97	0.20	0.034	0.013
UK	0.37	2.9	3.0	4.84	1.45	0.71	0.02	2.29	0.73	0.39	0.02	0.033	0.015
Europ. Avg	0.35	3.5	3.7	6.05	1.90	1.05	0.05	2.76	0.95	0.56	0.04	0.034	0.015

We compute the welfare losses in percent from a one percent technology shock ($\sigma_Z = 0.01, \rho_Z = 0.95$). We follow Lucas (2003) and estimate the utility from a deterministic consumption path and a risky consumption path with the same mean. We then calculate the amount of consumption necessary to make a risk averse household indifferent between the deterministic and risky consumption streams. The result is the loss in permanent consumption in percentage points for four scenarios: autarky, incomplete markets, complete markets, and a transfer union. There is no steady state terms of trade markup here. Openness (α) is taken from Balta and Delgado (2009). The elasticity of substitution between home and foreign products (η) and the elasticity of substitution between the products of different countries (γ) for European countries is taken from Table 4 and 5 of Corbo and Osbat (2013).

Table 3: Losses in Permanent Consumption for Country-Specific Elasticity Estimates of Imbs and Méjean (2010)

	Parameters			**High Wage Rigidity ($\theta_W = 0.87$)**				**Low Wage Rigidity ($\theta_W = 0.75$)**				**Flex Wages ($\theta_W = 0$)**	
	α	η	γ	Fin. Autarky	Incomp. Markets	Complete Markets	Transfer Union	Fin. Autarky	Incomp. Markets	Complete Markets	Transfer Union	Fin. Autarky	Incomp. Markets
Austria	0.55	1.9	3.9	4.94	2.21	1.50	0.18	2.33	1.10	0.79	0.16	0.031	0.012
Finland	0.32	3.5	3.4	5.81	1.63	0.81	0.02	2.67	0.81	0.44	0.02	0.034	0.016
France	0.31	3.1	3.5	5.70	1.53	0.72	0.01	2.63	0.76	0.39	0.01	0.035	0.016
Germany	0.34	3.5	3.8	6.19	1.91	1.05	0.04	2.81	0.95	0.56	0.04	0.035	0.015
Greece	0.35	4.8	4.2	7.22	2.48	1.53	0.12	3.18	1.22	0.80	0.11	0.035	0.015
Hungary	0.49	2.4	3.1	4.32	1.66	0.98	0.07	2.08	0.84	0.53	0.06	0.030	0.013
Italy	0.22	3.9	3.8	6.71	1.36	0.52	0.00	3.00	0.68	0.29	0.00	0.036	0.019
Portugal	0.41	3.6	4.9	6.99	2.73	1.82	0.19	3.10	1.33	0.95	0.18	0.035	0.014
Slovakia	0.30	3.2	2.7	4.99	1.19	0.47	0.00	2.35	0.60	0.26	0.00	0.034	0.017
Spain	0.30	3.5	4.4	6.80	1.95	1.04	0.03	3.03	0.97	0.56	0.03	0.036	0.016
Sweden	0.42	3.2	4.0	6.01	2.25	1.41	0.11	2.74	1.11	0.74	0.11	0.034	0.014
UK	0.37	3.2	3.6	5.65	1.82	1.00	0.04	2.61	0.91	0.54	0.04	0.034	0.015
European Avg	0.35	3.3	3.8	6.05	1.90	1.05	0.05	2.76	0.95	0.56	0.04	0.034	0.015

We compute the welfare losses in percent from a one percent technology shock ($\sigma_Z = 0.01, \rho_Z = 0.95$). We follow Lucas (2003) and estimate the utility from a deterministic consumption path and a risky consumption path with the same mean. We then calculate the amount of consumption necessary to make a risk averse household indifferent between the deterministic and risky consumption streams. The result is the loss in permanent consumption in percentage points for four scenarios: autarky, incomplete markets, complete markets, and a transfer union. There is no steady state terms of trade markup here. Openness (α) is taken from Balta and Delgado (2013). The elasticity of substitution between home and foreign products (η) and the elasticity of substitution between the products of different countries (γ) for European countries is taken from Imbs and Méjean (2010).

Table 4: Losses in Permanent Consumption in Incomplete Markets from Productivity Shocks Calibrated to the Data

	Data			**Corbo & Osbat (2013)**						**Imbs & Méjean (2010)**					
	α	$\sigma_{Y,1}$	$\sigma_{Y,2}$	η	γ	$\sigma_{Z,1}$	Losses	$\sigma_{Z,2}$	Losses	η	γ	$\sigma_{Z,1}$	Losses	$\sigma_{Z,2}$	Losses
Austria	0.55	0.013	0.005	4.5	3.8	0.010	1.087	0.004	0.147	1.9	3.9	0.011	0.982	0.004	0.132
Czech	0.70	0.019	0.010	3.4	3.8	0.015	2.065	0.007	0.514						
Denmark	0.40	0.015	0.007	3.3	3.4	0.013	1.318	0.006	0.291						
Finland	0.32	0.021	0.010	3.5	3.4	0.018	2.500	0.008	0.562	3.5	3.4	0.018	2.502	0.008	0.561
France	0.31	0.010	0.005	3.7	3.8	0.008	0.576	0.004	0.142	3.1	3.5	0.008	0.542	0.004	0.133
Germany	0.34	0.017	0.006	3.7	4.3	0.014	1.790	0.005	0.207	3.5	3.8	0.014	1.722	0.005	0.198
Greece	0.35	0.024	0.024	2.9	4.1	0.020	3.393	0.020	3.340	4.8	4.2	0.019	3.681	0.019	3.633
Hungary	0.49	0.017	0.010	3.3	4.2	0.013	1.767	0.008	0.629	2.4	3.1	0.014	1.458	0.008	0.521
Italy	0.22	0.014	0.004	3.2	3.2	0.012	0.954	0.003	0.079	3.9	3.8	0.011	1.050	0.003	0.087
Netherlands	0.62	0.013	0.005	3.5	3.5	0.010	0.987	0.004	0.141						
Portugal	0.41	0.013	0.011	3.3	3.9	0.010	1.006	0.009	0.750	3.6	4.9	0.010	1.081	0.009	0.811
Slovakia	0.30	0.023	0.014	3.7	3.9	0.019	3.020	0.012	1.200	3.2	2.7	0.019	2.600	0.012	1.033
Spain	0.30	0.012	0.007	3.4	3.2	0.010	0.848	0.006	0.291	3.5	4.4	0.010	0.941	0.006	0.323
Sweden	0.42	0.019	0.009	4.2	4.5	0.014	2.168	0.007	0.505	3.2	4.0	0.015	2.016	0.007	0.470
UK	0.37	0.014	0.008	2.9	3.0	0.011	0.936	0.006	0.299	3.2	3.6	0.011	1.030	0.006	0.326
European Avg	0.35	0.013	0.000	3.5	3.7	0.010	0.978	0.000	0.000	3.3	3.8	0.011	0.982	0.000	0.000

Here we compute the welfare losses in incomplete markets resulting from productivity shocks calibrated to match the volatility and autocorrelation of output in the data. The autocorrelation of HP-filtered output (ρ_Z) in the data is approximately 0.9998, so we set $\rho_Z = 0.99$ for the simulations above. The volatility of productivity shocks ($\sigma_{Z,1}$, $\sigma_{Z,2}$) are calibrated to match the volatilities of HP-filtered output ($\sigma_{Y,1}$, $\sigma_{Y,2}$). In Scenario 1 we take the volatility of output ($\sigma_{Y,1}$) from HP-filtered GDP data for each country with no adjustments and calibrate productivity shocks ($\sigma_{Z,1}$) to match the output volatilities from the data. In Scenario 2 we compute the volatility of output ($\sigma_{Y,2}$) from HP-filtered GDP data for each country after subtracting euro area GDP in order to construct a valid measure of asymmetric productivity shocks. We then calibrate the volatility of asymmetric productivity shocks ($\sigma_{Z,2}$) to match the volatility of asymmetric output in the data ($\sigma_{Y,2}$). We follow Lucas (2003) and estimate the utility from a deterministic consumption path and a risky consumption path with the same mean. We then calculate the amount of consumption necessary to make a risk averse household indifferent between the deterministic and risky consumption streams: the result is the losses in permanent consumption in percentage points. There is no steady state terms of trade markup here. Openness (α) is taken from Balta and Delgado (2009). The elasticity of substitution between home and foreign products (η) and the elasticity of substitution between the products of different countries (γ) for European countries is taken from Corbo and Osbat (2013) and from Imbs and Méjean (2010).

Figure 1: Welfare Losses from Business Cycle Fluctuations in the Closed-Form Model

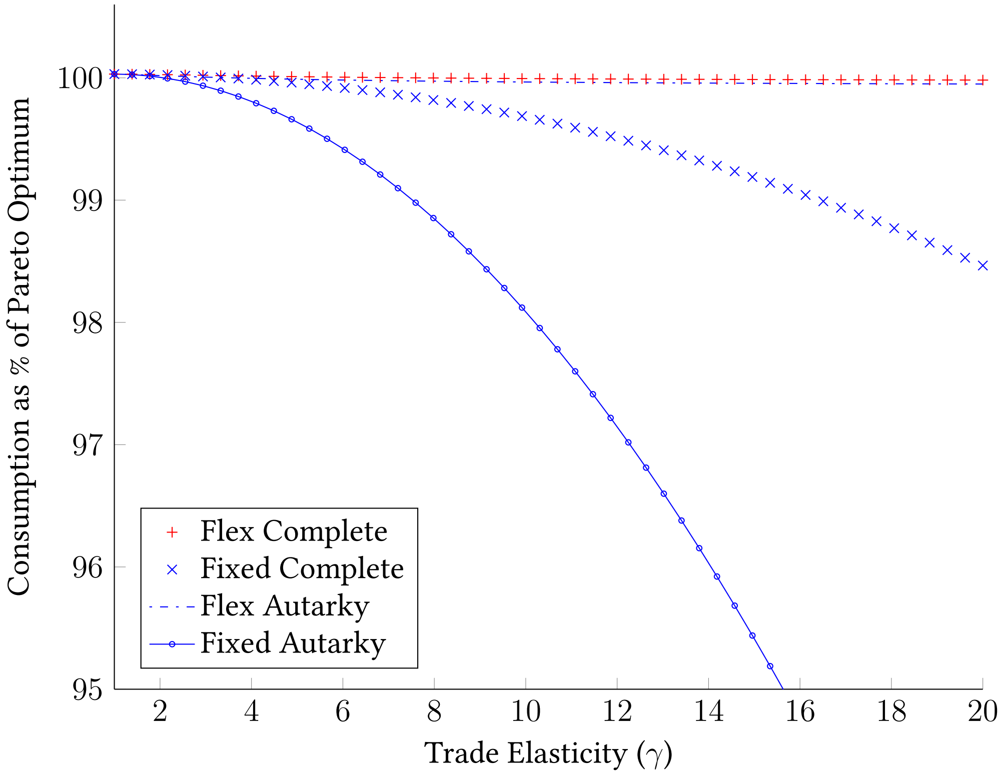


Figure 2: Welfare Losses from Business Cycle Fluctuations in Financial Autarky

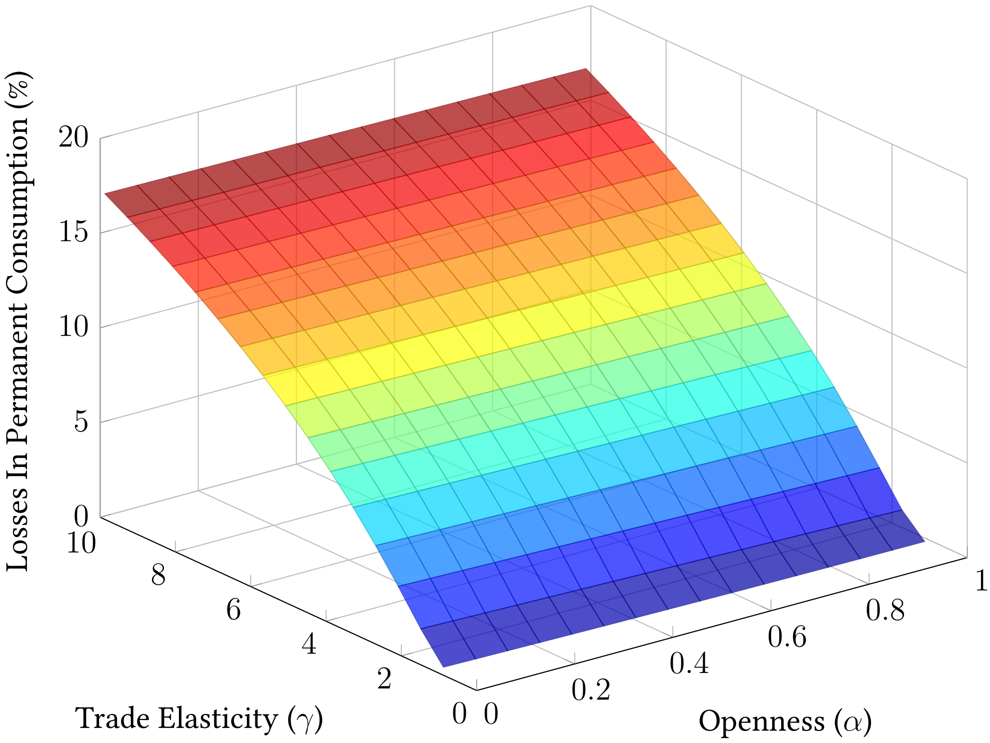


Figure 3: Welfare Losses from Business Cycle Fluctuations in Incomplete Markets

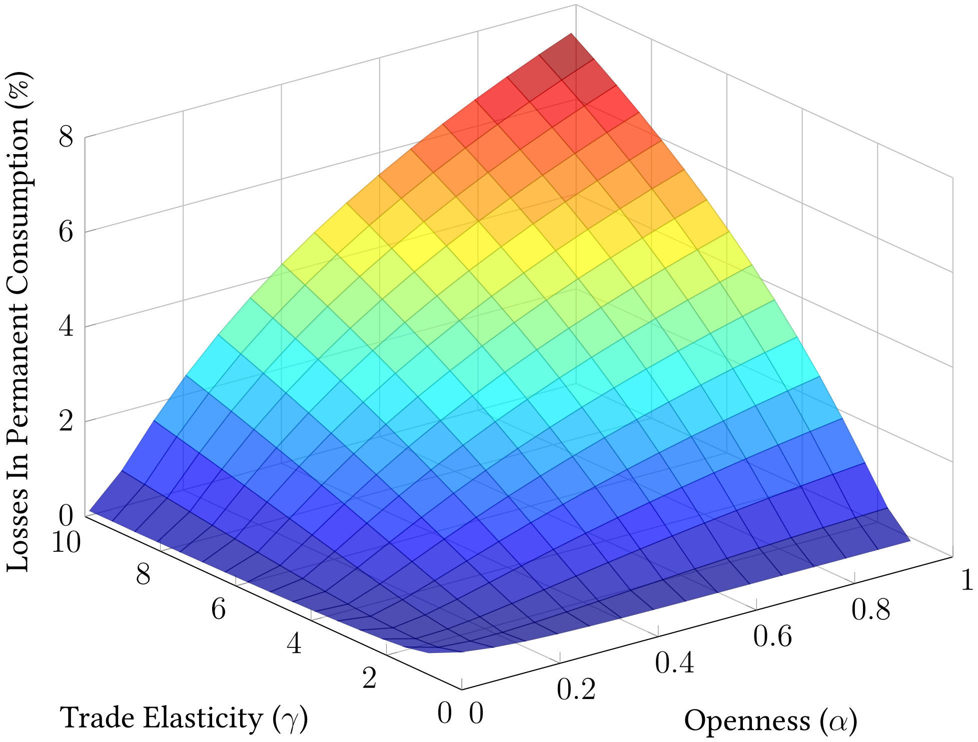


Figure 4: Welfare Losses from Business Cycle Fluctuations in a Transfer Union

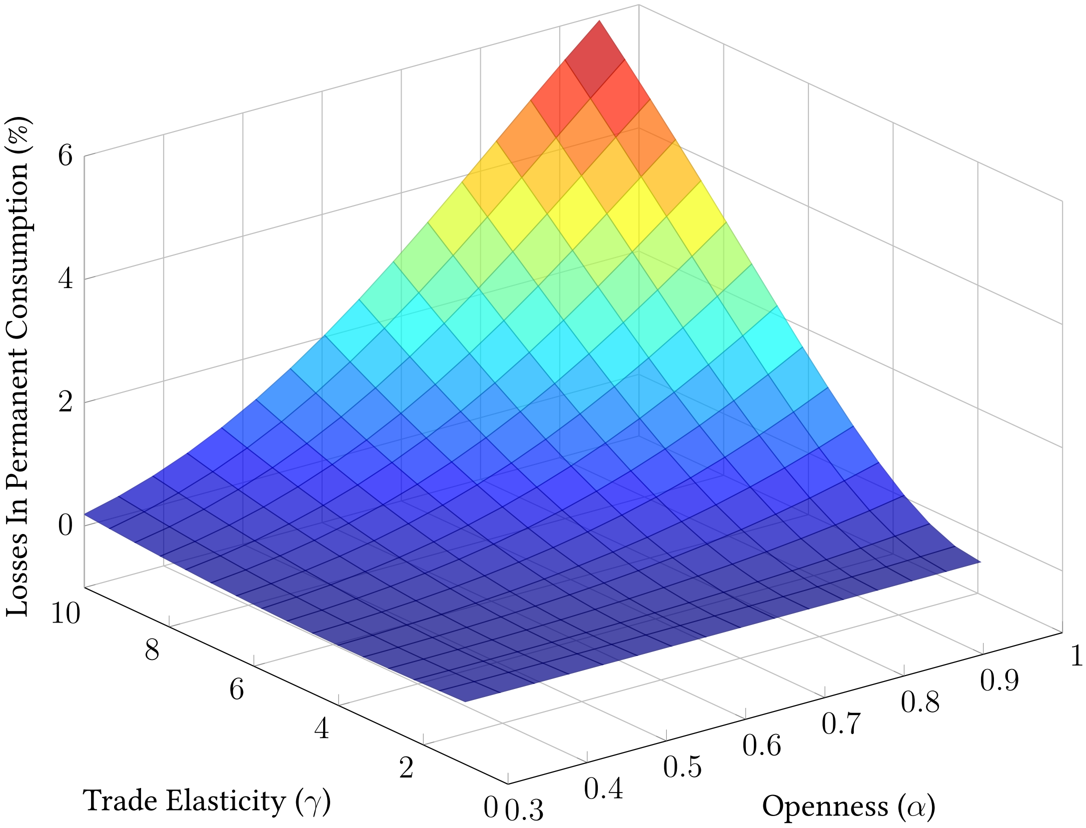
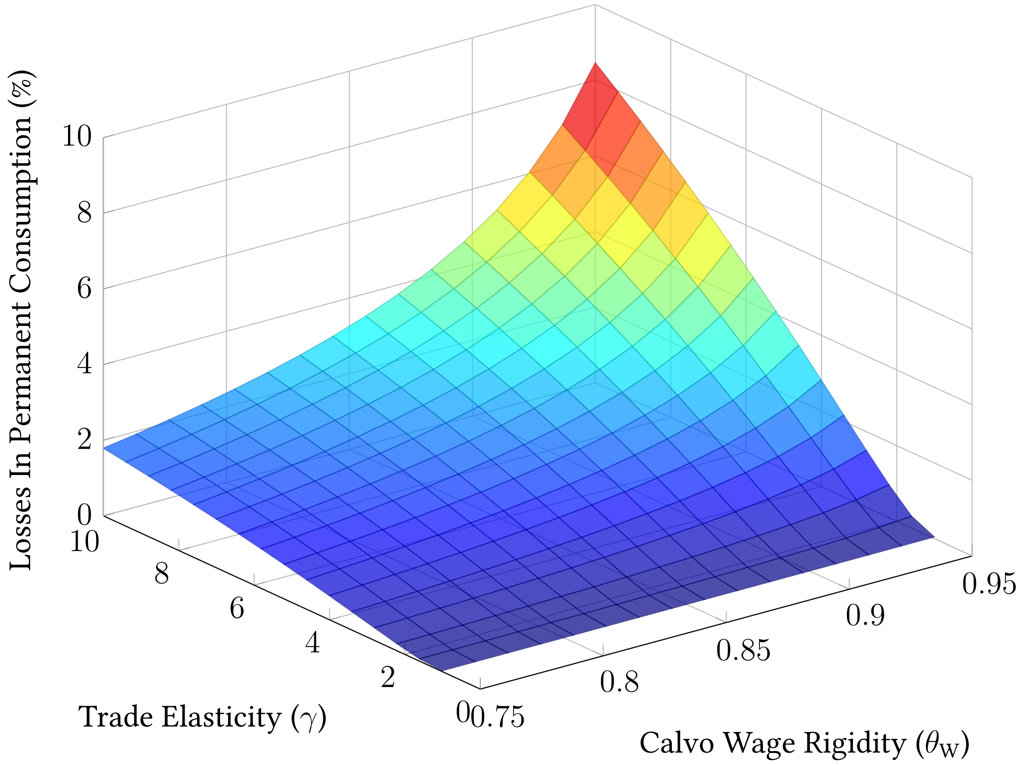


Figure 5: Welfare Losses from Business Cycle Fluctuations in Financial Autarky for Different Levels of Wage Rigidity



For Online Publication: Technical Appendix

A Proof of Main Propositions

A.1 Proof of Proposition 1: The Decentralized Flexible Wage, Complete Markets Allocation

The household in country i will maximize lifetime utility (3), subject to a simplified version of the household budget constraint and the transversality condition:

$$C_i(s_t)P_i(s_t) = W_i(s_t)N_i(s_t) + \mathcal{D}_i(s_t), \tag{A.1}$$

$$\mathbb{E}_0 \left\{ \sum_{t=1}^{\infty} q_i(s_t)\mathcal{D}_i(s_t) \right\} = 0. \tag{A.2}$$

$\mathcal{D}_{ij}(s_t)$ denotes the state-contingent bond that pays in currency j in state s_t; $q_j(s_t)$ is the price of that bond in period 0 (when all trading occurs), where $q_j(s_t)$ is arbitrary up to a constant. The household in period 0 cares about the relative price of claims across states and currencies. The transversality condition stipulates that all period 0 transactions must be balanced: payment for claims issued must equal payment for claims received. The household Lagrangian is:

$$\mathcal{L}_i = \sum_{t=1}^{\infty} \beta^t \mathbb{E}_0 \left\{ U_i(C(s_t)) - V_i(N(s_t)) + \frac{\lambda_i(s_t)}{P_i(s_t)} \left[W_i(s_t)N_i(s_t) + \mathcal{D}_i(s_t) - C_i(s_t)P_i(s_t) \right] \right\}$$
$$- \lambda_{i0} \sum_{t=1}^{\infty} \mathbb{E}_0 \big\{ q(s_t)\mathcal{D}_i(s_t) \big\}. \tag{A.3}$$

Now take the FOC with respect to state contingent bonds $\mathcal{D}_i(s_t)$:

$$\frac{\partial \mathcal{L}_i}{\partial D_i(s_t)} = \lambda_{i0} q(s_t) + \frac{\beta^t \lambda_i(s_t)}{P_i(s_t)} = 0, \tag{A.4}$$

which gives the price of the state-contingent bond,

$$q_i(s_t) = \beta^t \frac{\lambda_i(s_t)}{\lambda_{i0}} \frac{1}{P_i(s_t)}.$$

Substituting the above equation into (A.2) we can express the transversality condition as equation (5) in the text, which for reference sake is below:

$$\mathbb{E}_0 \left\{ \sum_{t=1}^{\infty} \beta^t \frac{C_{it}^{-\sigma}}{P_{it}} \mathcal{D}_{it} \right\} = 0.$$

Notice that the price of $q_i(s_t)$ does not depend on country i. In this case we get the risk-sharing condition

$$\frac{\lambda_i(s_t)}{\lambda_{i0}} \frac{1}{P_i(s_t)} = \frac{\lambda_j(s_t)}{\lambda_{j0}} \frac{1}{P_j(s_t)}, \tag{A.5}$$

which implies

$$\frac{C_i^{-\sigma}(s_t)}{P_i(s_t)} = constant.$$

When PPP holds (e.g. when there is no home bias), $\frac{P_i(s_t)}{P_j(s_t)} = 1$, and the risk-sharing condition simplifies to $\frac{\lambda_i(s_t)}{\lambda_j(s_t)} = \left(\frac{C_i(s_t)}{C_j(s_t)}\right)^{-\sigma} = \frac{\lambda_{i0}}{\lambda_{j0}}$. When the consumption ratio is constant across countries, $C_{it} = A_i P_i(s_t)^{-\frac{1}{\sigma}}$.

In order to solve for A_i, we substitute (A.1) into the transversality condition,

$$\sum_{t=1}^{\infty} \mathbb{E}_0\{q(s_t)\mathcal{D}_i(s_t)\} = \sum_{t=1}^{\infty} \mathbb{E}_0\left\{\beta^t \frac{C_i(s_t)^{-\sigma}}{P_i(s_t)}\big(C_i(s_t)P_i(s_t) - W_i(s_t)N_i(s_t)\big)\right\} = 0$$

and simplify the above expression to obtain

$$\sum_{t=1}^{\infty} \beta^t \mathbb{E}_0\left\{C_i^{1-\sigma}(s_t)\right\} = \sum_{t=1}^{\infty} \beta^t \mathbb{E}_0\left\{C_i(s_t)^{-\sigma}\frac{W_i(s_t)}{P_i(s_t)}N_i(s_t)\right\}.$$

Now we can extract and cancel out $C_i^{-\sigma}(s_t)P_i(s_t)$:

$$\sum_{t=1}^{\infty} \beta^t \mathbb{E}_0\left\{C_i(s_t)P_i(s_t)\right\} = \sum_{t=1}^{\infty} \beta^t \mathbb{E}_0\left\{W_i(s_t)N_i(s_t)\right\}.$$

Since $C_i = AP_i^{-\frac{1}{\sigma}}$, we can plug it into the transversality condition and obtain

$$C_i = \frac{\sum_{t=1}^{\infty} \beta^t \mathbb{E}_0\left\{W_i(s_t)N_i(s_t)\right\}}{\sum_{t=1}^{\infty} \beta^t \mathbb{E}_0\left\{P_i(s_t)^{\frac{\sigma-1}{\sigma}}\right\}} P_i^{-\frac{1}{\sigma}} = \frac{\sum_{t=1}^{\infty} \beta^t \mathbb{E}_0\left\{Y_i(s_t)P_{H,i}(s_t)\right\}}{\sum_{t=1}^{\infty} \beta^t \mathbb{E}_0\left\{P_i(s_t)^{\frac{\sigma-1}{\sigma}}\right\}} P_i^{-\frac{1}{\sigma}}, \tag{A.6}$$

which can be simplified to (19).

To solve for (21a), the household maximizes lifetime utility (3) subject to the budget constraint (4) and the labor demand condition (2). The solution to the household's constrained maximization problem is:

$$N_i^{\varphi} = \chi \frac{W_i}{P_i} C_i^{-\sigma}. \tag{A.7}$$

Using the production function (1) and the expression for the domestic price level in terms of the nominal wage and productivity (15), we obtain the simplified expression

$$Y_i^{\varphi} Z_i^{1-\varphi} = \chi C_i^{-\sigma} \frac{P_{H,i}}{P_i}. \tag{A.8}$$

■

A.2 Proof of Corollary 1: The Pareto Optimal Allocation

The social planner maximizes the sum of household utility across all i economies

$$\int_0^1 \left[\frac{C_i^{1-\sigma}}{1-\sigma} - \chi \frac{N_i^{1+\varphi}}{1+\varphi}\right] di \tag{A.9}$$

subject to the production constraint (1), the aggregate domestic consumption basket (6), the consumption index for foreign goods (7), and goods market clearing (12):

$$\begin{aligned}
Y_i &= Z_i N_i \\
C_i &= \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,i}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,i}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
C_{F,i} &= \left(\int_0^1 C_{F,ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}} \\
Y_i &= C_{H,i} + \int_0^1 C_{F,ji} dj.
\end{aligned}$$

We can plug in all of these constraints to the objective function and solve the following unconstrained maximization problem:

$$\max_{C_{H,i}, C_{Fij}} \int_0^1 \left[\frac{\left[(1-\alpha)^{\frac{1}{\eta}} C_{H,i}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(\int_0^1 C_{F,ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}\frac{\eta-1}{\eta}}\right]^{\frac{\eta(1-\sigma)}{\eta-1}}}{1-\sigma} - \chi \frac{\left(C_{H,i} + \int_0^1 C_{F,ji} dj\right)^{1+\varphi}}{Z_i^{1+\varphi}(1+\varphi)}\right] di. \tag{A.10}$$

The first order conditions with respect to $C_{F,ji}$ and $C_{H,i}$ are

$$\begin{aligned}
\frac{\partial U}{\partial C_{F,ji}} &= \alpha^{\frac{1}{\eta}} C_j^{-\sigma} C_j^{\frac{1}{\eta}} C_{F,j}^{-\frac{1}{\eta}} C_{F,j}^{\frac{1}{\gamma}} C_{F,ji}^{-\frac{1}{\gamma}} - \chi Y_i^{\varphi} Z_i^{-(1+\varphi)} = 0, \\
\frac{\partial U}{\partial C_{H,i}} &= (1-\alpha)^{\frac{1}{\eta}} C_i^{-\sigma} C_i^{\frac{1}{\eta}} C_{H,i}^{-\frac{1}{\eta}} - \chi Y_i^{\varphi} Z_i^{-(1+\varphi)} = 0.
\end{aligned}$$

We rearrange these equations by substituting out the term $\chi Y_i^{\varphi} Z_i^{-(1+\varphi)}$ and obtain:

$$\alpha^{\frac{1}{\eta}} (1-\alpha)^{-\frac{1}{\eta}} C_j^{-\sigma} C_j^{\frac{1}{\eta}} C_{F,j}^{-\frac{1}{\eta}} C_{F,j}^{\frac{1}{\gamma}} C_i^{\sigma} C_i^{-\frac{1}{\eta}} C_{H,i}^{\frac{1}{\eta}} = C_{F,ji}^{\frac{1}{\gamma}}. \tag{A.11}$$

Now we integrate the left hand side and the right hand side of the above equation across i by using $C_{F,j} = (\int_0^1 C_{F,ji}^{\frac{\gamma-1}{\gamma}} di)^{\frac{\gamma}{\gamma-1}}$:

$$\alpha^{\frac{1}{\eta}} (1-\alpha)^{-\frac{1}{\eta}} C_j^{-\sigma} C_j^{\frac{1}{\eta}} C_{F,j}^{-\frac{1}{\eta}} C_{F,j}^{\frac{1}{\gamma}} \left(\int_0^1 C_i^{\sigma\frac{\gamma-1}{\gamma}} C_i^{-\frac{1}{\eta}\frac{\gamma-1}{\gamma}} C_{H,i}^{\frac{1}{\eta}\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}} = C_{F,j}^{\frac{1}{\gamma}}.$$

The above is equivalent to the complete markets condition:

$$C_j^{\sigma} \alpha^{-\frac{1}{\eta}} C_j^{-\frac{1}{\eta}} C_{F,j}^{\frac{1}{\eta}} = (1-\alpha)^{-\frac{1}{\eta}} \left(\int_0^1 C_i^{\sigma\frac{\gamma-1}{\gamma}} C_i^{-\frac{1}{\eta}\frac{\gamma-1}{\gamma}} C_{H,i}^{\frac{1}{\eta}\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}}. \tag{A.12}$$

Now, we derive the expression for total exports from country i: $\int_0^1 C_{F,ji} dj$. To find $C_{F,ji}$ we use

(A.11) to obtain

$$C_{F,ji}^{\frac{1}{\gamma}} = (1-\alpha)^{-\frac{1}{\eta}} C_i^{\sigma} C_i^{-\frac{1}{\eta}} C_{H,i}^{\frac{1}{\eta}} \alpha^{\frac{1}{\eta}} C_j^{-\sigma} C_j^{\frac{1}{\eta}} C_{F,j}^{-\frac{1}{\eta}} C_{F,j}^{\frac{1}{\gamma}}. \tag{A.13}$$

Canceling out terms using the independence of $C_j^{-\sigma} C_j^{\frac{1}{\eta}} C_{F,j}^{-\frac{1}{\eta}}$ from country j, we have

$$C_{F,ji}^{\frac{1}{\gamma}} = (1-\alpha)^{-\frac{1}{\eta}} C_{F,i}^{-\frac{1}{\eta}} C_{H,i}^{\frac{1}{\eta}} \alpha^{\frac{1}{\eta}} C_{F,j}^{\frac{1}{\gamma}}. \tag{A.14}$$

Take both sides of equation to the power of γ and obtain

$$C_{F,ji} = (1-\alpha)^{-\frac{\gamma}{\eta}} C_{F,i}^{-\frac{\gamma}{\eta}} C_{H,i}^{\frac{\gamma}{\eta}} \alpha^{\frac{\gamma}{\eta}} C_{F,j}. \tag{A.15}$$

We compute exports from country i by integrating across the rest of the world consumption $\int_0^1 C_{F,ji} dj$:

$$\int_0^1 C_{F,ji} dj = (1-\alpha)^{-\frac{\gamma}{\eta}} C_{F,i}^{-\frac{\gamma}{\eta}} C_{H,i}^{\frac{\gamma}{\eta}} \alpha^{\frac{\gamma}{\eta}} \int_0^1 C_{F,j} dj. \tag{A.16}$$

The equilibrium is characterized by the four equations below:

$$Y_i = C_{H,i} + (1-\alpha)^{-\frac{\gamma}{\eta}} C_{F,i}^{-\frac{\gamma}{\eta}} C_{H,i}^{\frac{\gamma}{\eta}} \alpha^{\frac{\gamma}{\eta}} \int_0^1 C_{F,j} dj, \tag{A.17}$$

$$C_i = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,i}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,i}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{A.18}$$

$$\chi Y_i^{\varphi} Z_i^{1+\varphi} = (1-\alpha)^{\frac{1}{\eta}} C_i^{-\sigma} C_i^{\frac{1}{\eta}} C_{H,i}^{-\frac{1}{\eta}}, \tag{A.19}$$

$$C_j^{\sigma} \alpha^{-\frac{1}{\eta}} C_j^{-\frac{1}{\eta}} C_{F,j}^{\frac{1}{\eta}} = (1-\alpha)^{-\frac{1}{\eta}} \left(\int_0^1 C_i^{\sigma \frac{\gamma-1}{\gamma}} C_i^{-\frac{1}{\eta}\frac{\gamma-1}{\gamma}} C_{H,i}^{\frac{1}{\eta}\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}. \tag{A.20}$$

To show that this allocation exactly corresponds to the flexible wage, complete markets allocation given in (21b) and (21a), we need to make corresponding changes from the flexible wage allocation:

$$C_{H,i} = (1-\alpha)(P_{H,i}/P_i)^{-\eta} C_i, \tag{A.21}$$

$$C_{F,i} = \alpha(P_{F,i}/P_i)^{-\eta} C_i. \tag{A.22}$$

Now we are ready to make the necessary substitution and obtain:

$$Y_i = C_{H,i} + (P_{H,i}/P_{F,i})^{-\gamma} \int_0^1 C_{F,j} dj, \tag{A.23}$$

$$P_i^{1-\eta} = (1-\alpha) P_{H,i}^{1-\eta} + \alpha P_{F,i}^{1-\eta}, \tag{A.24}$$

$$\chi Y_i^{\varphi} Z_i^{1+\varphi} = C_i^{-\sigma} (P_{H,i}/P_i), \tag{A.25}$$

$$C_j^{\sigma} P_j = (1-\alpha)^{-\frac{1}{\eta}} \left(\int_0^1 C_i^{\sigma \frac{\gamma-1}{\gamma}} P_i^{\frac{\gamma-1}{\gamma}} P_{H,i}^{-\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} = constant, \tag{A.26}$$

which exactly corresponds to the flexible wage allocation given by (21a) and (21b).

A.3 Proof of Proposition 2: The Decentralized Sticky Wage, Incomplete Markets Allocation

As in the text, we omit country i and time t subindices for the *nominal* wage W which is set one-period-in-advance, as it will be identical across countries and across time. Goods market clearing takes the following form:

$$Y_{it} = (1-\alpha)C_{it}\left(\frac{P_{H,it}}{P_{it}}\right)^{-\eta} + \alpha W^{-\gamma} Z_{it}^{\gamma}\mathbb{E}_{t-1}\{C_{it}P_{it}^{\eta}\}. \tag{A.27}$$

In a monetary union, domestic exports from country i to the rest of the union are equal to

$$\alpha P_{H,it}^{-\gamma}\int_0^1 C_{F,j}dj = \alpha W^{-\gamma}Z_{it}^{\gamma}\int_0^1 C_{it}P_{it}^{\eta}di = \alpha W^{-\gamma}Z_{it}^{\gamma}\mathbb{E}_{t-1}\{C_{it}P_{it}^{\eta}\}.$$

Using the normalized foreign price index ($P_{F,t} = 1$) we can rewrite the aggregate domestic price index (15) as

$$P_{it}^{1-\eta} = (1-\alpha)P_{H,it}^{1-\eta} + \alpha, \tag{A.28}$$

and using (A.28), the price of the domestic variety (15) and the aggregate resource constraint under imperfect cross-country risk-sharing from (20), we can rewrite goods market clearing from (A.27) as

$$Y_{it} = (1-\alpha)Y_{it}\left(\frac{P_{H,it}}{P_{it}}\right)^{1-\eta} + (1-\alpha)\frac{\mathcal{T}_{it}}{P_{it}}\left(\frac{P_{H,it}}{P_{it}}\right)^{-\eta} + \alpha P_{H,it}^{-\gamma}\mathbb{E}_{t-1}\{C_{it}P_{it}^{\eta}\}.$$

Isolating output on the left hand side

$$Y_{it} = \frac{(1-\alpha)\frac{\mathcal{T}_{it}}{P_{it}}\left(\frac{P_{H,it}}{P_{it}}\right)^{-\eta} + \alpha P_{H,it}^{-\gamma}\mathbb{E}_{t-1}\{C_{it}P_{it}^{\eta}\}}{1-(1-\alpha)\left(\frac{P_{H,it}}{P_{it}}\right)^{1-\eta}},$$

we simplify the above expression and obtain

$$Y_{it} = P_{H,it}^{-\gamma}P_{it}^{1-\eta}\mathbb{E}_{t-1}\{C_{it}P_{it}^{\eta}\} + \frac{1-\alpha}{\alpha}P_{H,it}^{-\eta}\mathcal{T}_{it}. \tag{A.29}$$

From (A.29) we solve for labor using domestic production (1) to substitute out Y_{it} and we solve for consumption using the aggregate resource constraint from (20) to substitute out Y_{it}:

$$N_{it} = P_{H,it}^{-\gamma}P_{it}^{1-\eta}Z_{it}^{-1}\mathbb{E}_{t-1}\{C_{it}P_{it}^{\eta}\} + \frac{1-\alpha}{\alpha}P_{H,it}^{-\eta}Z_{it}^{-1}\mathcal{T}_{it}, \tag{A.30}$$

$$C_{it} = P_{H,it}^{1-\gamma}P_{it}^{-\eta}\mathbb{E}_{t-1}\{C_{it}P_{it}^{\eta}\} + \frac{1-\alpha}{\alpha}P_{H,it}^{1-\eta}\frac{\mathcal{T}_{it}}{P_{it}} + \frac{\mathcal{T}_{it}}{P_{it}}. \tag{A.31}$$

Simplifying the above expression for consumption

$$C_{it} = P_{H,it}^{1-\gamma}P_{it}^{-\eta}\mathbb{E}_{t-1}\{C_{it}P_{it}^{\eta}\} + \frac{1}{\alpha}P_{it}^{-\eta}\mathcal{T}_{it}, \tag{A.32}$$

and solving for transfers in (A.32) yields:

$$\mathcal{T}_{it} = \alpha C_{it} P_{it}^{\eta} - \alpha P_{H,it}^{1-\gamma} \mathbb{E}_{t-1}\{C_{it} P_{it}^{\eta}\}. \tag{A.33}$$

We can also directly express labor as a function of consumption. In particular:

$$\frac{C_{it} - P_{H,it}^{1-\gamma} P_{it}^{-\eta} \mathbb{E}_{t-1}\{C_{it} P_{it}^{\eta}\}}{\frac{1}{\alpha} P_{it}^{-\eta}} = \frac{N_{it} - P_{H,it}^{-\gamma} P_{it}^{1-\eta} Z_{it}^{-1} \mathbb{E}_{t-1}\{C_{it} P_{it}^{\eta}\}}{\frac{1-\alpha}{\alpha} P_{H,it}^{-\eta} Z_{it}^{-1}}. \tag{A.34}$$

After a few algebraic manipulations we get

$$N_{it} = (C_{it} - P_{H,it}^{1-\gamma} P_{it}^{-\eta} \mathbb{E}_{t-1}\{C_{it} P_{it}^{\eta}\})(1-\alpha) P_{H,it}^{-\eta} Z_{it}^{-1} P_{it}^{\eta} + P_{H,it}^{-\gamma} P_{it}^{1-\eta} Z_{it}^{-1} \mathbb{E}_{t-1}\{C_{it} P_{it}^{\eta}\}. \tag{A.35}$$

which can be simplified to:

$$N_{it} = (1-\alpha) P_{H,it}^{-\eta} Z_{it}^{-1} P_{it}^{\eta} C_{it} + \alpha P_{H,it}^{-\gamma} Z_{it}^{-1} \mathbb{E}_{t-1}\{C_{it} P_{it}^{\eta}\}.. \tag{A.36}$$

A.4 Proof of Proposition 3: Optimal Transfers

In what follows we make use of the following relationships which hold in a monetary union:

$$\int_0^1 C_{F,jit} dj = (P_{H,it}/P_{F,t})^{-\gamma} \int_0^1 C_{F,jt} dj = (P_{H,it}/P_{F,t})^{-\gamma} \alpha \int_0^1 C_{jt} (P_{F,t}/P_{jt})^{-\eta} dj = \alpha P_{H,it}^{-\gamma} \mathbb{E}_{t-1}\{C_{jt} P_{jt}^{\eta}\}.$$

In this section we also omit the subindex i. We combine the consumption allocation (27a) and labor allocation (27b) from the decentralized, incomplete markets, stick wage equilibrium into (A.36). We then formulate the policymaker's Lagrangian for the optimal transfer scheme:

$$\begin{aligned} \mathcal{L} =& \mathbb{E}_0 \Bigg\{ \sum_{t=1}^{\infty} \beta^t \Bigg[\frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} + \lambda_{it} \Big(N_{it} - (1-\alpha) P_{H,it}^{-\eta} Z_{it}^{-1} P_{it}^{\eta} C_{it} - \alpha P_{H,it}^{-\gamma} Z_{it}^{-1} \mathbb{E}_{t-1}\{C_{it} P_{it}^{\eta}\} \Big) \\ & - \lambda_W \Big(\chi \mathbb{E}_{t-1}\{N_{it}^{1+\varphi}\} - (1-\alpha) \mathbb{E}_{t-1}\{C_{it}^{1-\sigma} P_{H,it}^{1-\eta} P_{it}^{\eta-1}\} - \alpha \mathbb{E}_{t-1}\{C_{it}^{-\sigma} P_{H,it}^{1-\gamma} P_{it}^{-1}\} \mathbb{E}_{t-1}\{C_{it} P_{it}^{\eta}\} \Big) \Bigg] \Bigg\}. \end{aligned} \tag{A.37}$$

The first order conditions with respect to consumption and labor are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_{it}} =& C_{it}^{-\sigma} - \lambda_{it}(1-\alpha) P_{H,it}^{-\eta} Z_{it}^{-1} P_{it}^{\eta} - \alpha \mathbb{E}_{t-1}\{\lambda_{it} P_{H,it}^{-\gamma} Z_{it}^{-1}\} P_{it}^{\eta} - \lambda_W (1-\alpha)(1-\sigma) C_{it}^{-\sigma} P_{H,it}^{1-\eta} P_{it}^{\eta-1} \\ & + \alpha \lambda_W \sigma C_{it}^{-\sigma-1} P_{H,it}^{1-\gamma} P_{it}^{-1} \mathbb{E}_{t-1}\{C_{it} P_{it}^{\eta}\} - \alpha \lambda_W \mathbb{E}_{t-1}\{C_{it}^{-\sigma} P_{H,it}^{1-\gamma} P_{it}^{-1}\} P_{it}^{\eta} = 0, \end{aligned} \tag{A.38}$$

$$\frac{\partial \mathcal{L}}{\partial N_{it}} = -\chi N_{it}^{\varphi} + \lambda_{it} - \lambda_W \chi (1+\varphi) N_{it}^{\varphi} = 0. \tag{A.39}$$

We solve for λ_{it} using the first order condition for labor (A.39), and substitute into the first order condition for consumption:

$$\begin{aligned} 0 =& C_{it}^{-\sigma} - \chi(1 + \lambda_W(1+\varphi)) N_{it}^{\varphi} (1-\alpha) P_{H,it}^{-\eta} Z_{it}^{-1} P_{it}^{\eta} \\ & - \alpha \chi (1 + \lambda_W(1+\varphi)) E(N_{it}^{\varphi} P_{H,it}^{-\gamma} Z_{it}^{-1}) P_{it}^{\eta} - \lambda_W (1-\alpha)(1-\sigma) C_{it}^{-\sigma} P_{H,it}^{1-\eta} P_{it}^{\eta-1} \\ & + \alpha \lambda_W \sigma C_{it}^{-\sigma-1} P_{H,it}^{1-\gamma} P_{it}^{-1} E(C_{it} P_{it}^{\eta}) - \alpha \lambda_W E(C_{it}^{-\sigma} P_{H,it}^{1-\gamma} P_{it}^{-1}) P_{it}^{\eta}. \end{aligned} \tag{A.40}$$

At the steady state the above equation becomes

$$0 = C^{-\sigma} - \chi(1+\lambda_W(1+\varphi))N^{\varphi}(1-\alpha) - \alpha\chi(1+\lambda_W(1+\varphi))N^{\varphi} \\ - \lambda_W(1-\alpha)(1-\sigma)C^{-\sigma} + \alpha\lambda_W\sigma C^{-\sigma} - \alpha\lambda_W C^{-\sigma}. \tag{A.41}$$

We simplify the above expression to

$$0 = C^{-\sigma}(1-\lambda_W(1-\sigma)) - \chi(1+\lambda_W(1+\varphi))N^{\varphi}. \tag{A.42}$$

and use the fact that $C^{1-\sigma} = \chi N^{1+\varphi}$ and thus $N = C$ in steady state:

$$1 + \lambda_W(1+\varphi) = 1 - \lambda_W(1-\sigma). \tag{A.43}$$

Correspondingly, $\lambda_W = 0$. This allows us to simplify the first order condition for consumption:

$$0 = C_{it}^{-\sigma} - \chi N_{it}^{\varphi}(1-\alpha)P_{H,it}^{-\eta}Z_{it}^{-1}P_{it}^{\eta} - \alpha\chi E(N_{it}^{\varphi}P_{H,it}^{-\gamma}Z_{it}^{-1})P_{it}^{\eta}. \tag{A.44}$$

We log-linearize the simplified first order condition for consumption (A.44) and use the fact that expectations of all variables are equal to zero due to the independence of idiosyncratic shocks across time and space. In particular,

$$0 = -\sigma C^{-\sigma}\hat{C}_{it} - \chi N^{\varphi}(1-\alpha)(\varphi\hat{N}_{it} - \eta\hat{P}_{H,it} - \hat{Z}_{it} + \eta\hat{P}_{it}) - \alpha\chi N^{\varphi}\eta\hat{P}_{it}. \tag{A.45}$$

We again use the steady state relationship $C^{1-\sigma} = \chi N^{1+\varphi}$ and $N = C$, such that, $C^{-\sigma} = \chi N^{\varphi}$. Equation (A.45) simplifies to

$$0 = -\sigma\hat{C}_{it} - (1-\alpha)(\varphi\hat{N}_{it} - \eta\hat{P}_{H,it} - \hat{Z}_{it} + \eta\hat{P}_{it}) - \alpha\eta\hat{P}_{it}, \tag{A.46}$$

We simplify further and obtain

$$0 = \sigma\hat{C}_{it} + (1-\alpha)\varphi\hat{N}_{it} - (1-\alpha)\hat{Z}_{it}, \tag{A.47}$$

which is equivalent to the linear combination of the labor and consumption wedges:

$$(1-\alpha)\hat{V}_{N,it} + \alpha\hat{V}_{C,it} = 0. \tag{A.48}$$

■

B Global Closed-form Solution Under Zero Home Bias

B.1 Deriving the Allocations with Zero Home Bias

In this section we solve the model globally in closed-form for economies with zero home bias, where $1 - \alpha = 0$. The global closed-form solution allows us to derive explicit expressions for welfare that hold in any steady state, without any log-linearization.

Flexible Wages with Complete Markets: the Pareto Optimal Allocation

If we take the domestic consumption basket (6) to the limit as $\alpha \to 1$ and substitute the result along with domestic production (1) directly into the household objective function (3), then we can reformulate the policymaker's optimization problem as follows:[10]

$$\max_{\forall c_{ij}} \int_0^1 \left[\frac{\left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma(1-\sigma)}{\gamma-1}}}{1-\sigma} - \frac{\chi}{1+\varphi} \left(\frac{\int_0^1 c_{ji} dj}{Z_i} \right)^{1+\varphi} \right] di. \tag{B.1}$$

The first order condition with respect to c_{ij} is

$$0 = \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma(1-\sigma)}{\gamma-1}-1} c_{ij}^{-\frac{1}{\gamma}} - \chi \frac{\left(\int_0^1 c_{ji} dj\right)^{\varphi}}{Z_j^{1+\varphi}}. \tag{B.2}$$

This is equivalent to

$$0 = \underbrace{\left[\left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{1-\sigma\gamma}{\gamma-1}}\right]}_{=C_i^{\frac{1-\sigma\gamma}{\gamma}}} c_{ij}^{-\frac{1}{\gamma}} - \chi \underbrace{\left(\frac{\int_0^1 c_{ji} dj}{Z_j}\right)^{\varphi}}_{=N_j^{\varphi}} \frac{1}{Z_j} \Rightarrow 0 = C_i^{\frac{1-\sigma\gamma}{\gamma}} c_{ij}^{-\frac{1}{\gamma}} - \chi \frac{N_j^{\varphi}}{Z_j},$$

and solving for c_{ij} we have:

$$c_{ij} = \frac{Z_j^{\gamma} C_i^{1-\gamma\sigma}}{\chi^{\gamma} N_j^{\gamma\varphi}}. \tag{B.3}$$

The consumption basket in country i (C_i) can then be expressed as:

$$C_i = \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}} = \left[\int_0^1 \left(\frac{Z_j^{\gamma} C_i^{1-\gamma\sigma}}{\chi^{\gamma} N_j^{\gamma\varphi}}\right)^{\frac{\gamma-1}{\gamma}} dj\right]^{\frac{\gamma}{\gamma-1}} = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma}} \left[\int_0^1 \left(\frac{Z_j}{N_j^{\varphi}}\right)^{(\gamma-1)} dj\right]^{\frac{1}{\sigma(\gamma-1)}}. \tag{B.4}$$

So C_i does not depend on its own technology Z_i. Now, let's solve for labor N_i and output Y_i.

$$N_i = \frac{Y_i}{Z_i} = \frac{\int_0^1 c_{ji} dj}{Z_i} = \frac{\int_0^1 \left(\frac{Z_i^{\gamma} C_j^{1-\gamma\sigma}}{\chi^{\gamma} N_i^{\gamma\varphi}}\right) dj}{Z_i} = \frac{Z_i^{\gamma-1}}{\chi^{\gamma} N_i^{\gamma\varphi}} \int_0^1 C_j^{1-\gamma\sigma} dj \tag{B.5}$$

Since $C_i = C_j = C$ for all i, j. So we can take C_j outside of the integral in (B.5) and solve for N_i:

$$N_i = \frac{Z_i^{\gamma-1} C_j^{1-\gamma\sigma}}{\chi^{\gamma} N_i^{\gamma\varphi}} \Rightarrow N_i = \left(\frac{Z_i^{\gamma-1} C^{1-\gamma\sigma}}{\chi^{\gamma}}\right)^{\frac{1}{1+\gamma\varphi}}. \tag{B.6}$$

[10]Note that this policymaker could be a central bank in a flexible exchange rate regime.

Substitute (B.6) back into the definition of the consumption basket (B.4), and solve for the consumption basket C in each country, which will be identical:

$$C = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma}} \left\{ \int_0^1 \left[\left(\frac{Z_j^{\gamma-1} C^{1-\gamma\sigma}}{\chi^\gamma} \right)^{\frac{1}{1+\gamma\varphi}} \right]^{-(\gamma-1)\varphi} Z_j^{\gamma-1} dj \right\}^{\frac{1}{\sigma(\gamma-1)}},$$

$$\Rightarrow C = C_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} dj \right)^{\frac{1+\gamma\varphi}{(\sigma+\varphi)(\gamma-1)}}. \tag{B.7}$$

Solve for labor and output by substituting (B.7) into (B.6) and $Y_i = N_i Z_i$ respectively:

$$N_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} dj \right)^{\frac{1-\gamma\sigma}{(\sigma+\varphi)(\gamma-1)}} Z_i^{\frac{\gamma-1}{1+\gamma\varphi}}, \tag{B.8}$$

$$Y_i = \left(\frac{1}{\chi}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} dj \right)^{\frac{1-\gamma\sigma}{(\sigma+\varphi)(\gamma-1)}} Z_i^{\frac{\gamma(1+\varphi)}{1+\gamma\varphi}}. \tag{B.9}$$

This is the Pareto efficient allocation. Under cooperative subsidies it is identical to the flexible wage allocation.

Sticky Wage Allocations With No Home Bias

From (11) with no home bias and (15) we can compute labor using $Y_{it} = Z_{it} N_{it}$:

$$N_{it} = A Z_{it}^{\gamma-1}. \tag{B.10}$$

Given the above, consumption will be

$$C_{it} = C_{wt} = A \left(\int_0^1 Z_{it}^{\gamma-1} di \right)^{\frac{\gamma}{\gamma-1}}. \tag{B.11}$$

Using labor market clearing (13), and substituting in Y_{it}, C_{it}, N_{it} expressed as functions of A and Z_{it} from above, we find:

$$1 = \left(\frac{\chi \mu_\varepsilon}{1-\tau_i} \right) \frac{A^{1+\varphi} \int_0^1 Z_{it}^{(\gamma-1)(1+\varphi)} di}{A^{1-\sigma} \left(\int_0^1 Z_{it}^{(\gamma-1)} di \right)^{\frac{\gamma(1-\sigma)}{\gamma-1}}} \tag{B.12}$$

Now we can solve for A:

$$A = \left(\frac{\chi \mu_\varepsilon}{1-\tau_i} \right)^{\frac{-1}{\sigma+\varphi}} \left(\int_0^1 Z_{it}^{(\gamma-1)(1+\varphi)} di \right)^{\frac{-1}{\sigma+\varphi}} \left(\int_0^1 Z_{it}^{\gamma-1} di \right)^{\frac{\gamma}{\gamma-1} \frac{(1-\sigma)}{\sigma+\varphi}}. \tag{B.13}$$

Given this solution for the constant A, one can solve for C_{it} and N_{it} by substituting A into the expressions above, resulting in (27a) for C_{it} and (27b) for N_{it}. The same exercise in financial autarky will yield the expressions for C_{it} and N_{it}. ■

B.2 Welfare Results

Below, we outline the steps necessary to derive the expected utility functions contained in Section 4 of the paper. Here we only conduct the exercise for flexible exchange rates in complete markets, but following the steps presented here will also yield the expected utility functions for the other allocations.

$$C_{flex,complete} = \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(\sigma+\varphi)}}$$

$$\mathbb{E}\left\{U_{flex,complete}\right\} = \left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] \mathbb{E}\Big\{C^{1-\sigma}_{flex,complete}\Big\}$$

$$= \left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1-\sigma}{\sigma+\varphi}} \mathbb{E}\left\{\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{(1+\gamma\varphi)(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}}\right\}$$

For normative analysis, we assume that technology is log-normally distributed and is independent across time and across countries: $\log Z_{it} \sim N(0, \sigma_Z^2)$. The expectation above can then be rewritten as:

$$\mathbb{E}\left\{\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{(1+\gamma\varphi)(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}}\right\} = e^{\left[\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}\right]^2 \frac{(1+\gamma\varphi)(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}\sigma_Z^2} = e^{\frac{(\gamma-1)(1+\varphi)^2(1-\sigma)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2}.$$

Now, we insert this expression back into the original equation and get:

$$\mathbb{E}\left\{U_{flex,complete}\right\} = \left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1-\sigma}{\sigma+\varphi}} e^{\frac{(\gamma-1)(1+\varphi)^2(1-\sigma)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2}.$$

Taking logarithms, we can rewrite the log of expected utility as:

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} = \log\left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] + \frac{1-\sigma}{\sigma+\varphi}\log\left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right) + \frac{(\gamma-1)(1+\varphi)^2(1-\sigma)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2. \tag{B.14}$$

Calculating the expected utility for the other coalitions simply requires that one follow the steps outlined here. Notice that when we calculate welfare differences between allocations, the first and second terms on the right hand side of equation (B.14) will cancel out, leaving only the difference between the remaining term on the right hand side.

Using the expected utilities from (31a) – (31d), and the fact that any constant terms will cancel out when subtracted from each other, we calculate the welfare differences for four scenarios: (1) complete markets vs. autarky for flexible wages; (2) complete markets vs. autarky for fixed wages; (3) flexible vs. fixed wages for complete markets; and (4) flexible vs. fixed wages for autarky. When comparing welfare across different allocations, it is important to keep in mind that as risk-aversion decreases, (i.e. as $\sigma \to 1$), the welfare differences expressed in logarithms also decrease but the *absolute values* of utility increase. In other words, when risk aversion is low, the welfare differences shown in (B.15a) – (B.15d) will shrink, but this does not mean that the welfare differences are decreasing in absolute value.

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{flex,autarky}\right\} = \frac{\sigma(\gamma-1)^2(1-\sigma)(1+\varphi)^2}{(\sigma+\varphi)(1+\gamma\varphi)[1-\sigma+\gamma(\sigma+\varphi)]}\sigma_Z^2 \quad \text{(B.15a)}$$

$$\log \mathbb{E}\left\{U_{fixed,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} = \frac{\sigma(\gamma-1)^2(1-\sigma)(1+\varphi)}{\sigma+\varphi}\sigma_Z^2 \quad \text{(B.15b)}$$

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,complete}\right\} = \frac{\gamma\varphi^2(\gamma-1)^2(1-\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2 \quad \text{(B.15c)}$$

$$\log \mathbb{E}\left\{U_{flex,autarky}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} = \frac{(\gamma-1)^2(1-\sigma)(1+\varphi)[\gamma(\sigma+\varphi)-\sigma]}{1+\gamma(\sigma+\varphi)-\sigma}\sigma_Z^2 \quad \text{(B.15d)}$$

C Extended Model With Calvo Wage Rigidity And Non-Contingent Bonds

The extended model is identical to that described in Section (2) with two key differences. First, Calvo wage setting replaces one-period-in-advance wage setting. Second, we consider an additional financial market setup wherein countries may trade non-contingent bonds. In all other respects the model remains unchanged: household utility is (3), the consumption basket is (6) and the import basket is defined in (7), the price index is (8), the relative demand for home and foreign products is (9) and (10) respectively, demand for country i's good is (11) and goods market clearing for country i's unique variety is (12).

The introduction of a nominal non-contingent bond, B_{it}, that pays in units of the import basket $C_{F,it}$ defined in (7), will alter the household budget constraint from (4). In the bond economy, households will maximize utility from (3) subject to the following budget constraint:

$$C_{it}(h) + \frac{B_{it}(h)}{P_{it}} = (1-\tau_i)\left(\frac{W_{it}(h)}{P_{it}(h)}\right)N_{it}(h) + \mathcal{D}_{it}(h) + \mathcal{T}_{it}(h) + \Gamma_{it}(h) + (1+i_{t-1})\left(\frac{B_{it-1}(h)}{P_{it}}\right). \quad \text{(C.1)}$$

Country i's domestic interest rate i_{it} equals the world interest rate i^* plus a country specific interest rate premium $p()$ that is strictly increasing in the amount of debt B_{it}:

$$i_{it} = i^* + p(B_{it}). \quad \text{(C.2)}$$

Financial autarky is the case for which p goes to infinity. The interest rate premium is necessary to ensure stationarity.

The formulas for consumption in complete markets (from (A.6)), the bond economy, financial autarky, and financial autarky under a transfer union are given below:

$$C_{it} = \frac{\mathbb{E}_0\left\{\sum \beta^t Y_{it} P_{H,it}\right\}}{\mathbb{E}_0\left\{\sum \beta^t \left(P_{it}\right)^{\frac{\sigma-1}{\sigma}}\right\}} P_{it}^{-\frac{1}{\sigma}} \quad \text{(C.3a)}$$

$$C_{it} = \frac{Y_{it}P_{H,it}}{P_{it}} - B_{it} + B_{it-1}(1+i_{it-1}) \quad \text{(C.3b)}$$

$$C_{it} = \frac{Y_{it}P_{H,it}}{P_{it}} \quad \text{(C.3c)}$$

$$C_{it} = \frac{Y_{it}P_{H,it}}{P_{it}} + \frac{\mathcal{T}_{it}}{P_{it}} \quad \text{(C.3d)}$$

Under Calvo, the aggregate wage in period t (W_{it}) is a weighted average of the optimal reset wage ($\tilde{W}_{it}$) and the previous period wage (W_{it-1}):

$$W_{it}^{1-\varepsilon} = (1-\theta_W)\tilde{W}_{it}^{1-\varepsilon} + \theta_W W_{it-1}^{1-\varepsilon} \tag{C.4}$$

where θ_W is the fraction of households who are able to reset wages in each period. V_W and $\tilde{V}_W$ are auxiliary variables that we use to describe infinite summations. The equations describing Calvo wage setting are:

$$V_{W,it} = N_{it}^{1+\varphi} + \beta\theta_W \mathbb{E}_t V_{W,it+1} \tag{C.5}$$

$$\tilde{V}_{W,it} = C_{it}^{-\sigma}\frac{N_{it}}{P_{it}} + \beta\theta_W \tilde{V}_{W,it+1} \tag{C.6}$$

$$\tilde{W}_{it} = \chi\frac{\varepsilon}{\varepsilon-1}\frac{V_{W,it}}{\tilde{V}_{W,it}} \tag{C.7}$$

$$W_{it}^{1-\varepsilon} = (1-\theta_W)\tilde{W}_{it}^{1-\varepsilon} + \theta_W W_{it-1}^{1-\varepsilon} \tag{C.8}$$

To compute welfare under financial autarky, incomplete markets, complete markets and a transfer union, we solve a second-order approximation of the model for each country assuming that the home country is a small open economy and that the rest of the union is in the steady state. In this case the calibration of parameters for other members of the union has no effect. The correct specification yields the ergodic mean for the rest of the union. We compute the ergodic mean (assuming that the rest of the union has an identical calibration to the small open economy, but faces asymmetric shocks) and find that it has no effect on optimal allocations or welfare analysis. Indeed, since we consider only asymmetric shocks, and in the aggregate positive shocks cancel out negative shocks across the union, it does not matter whether the rest of the union is at rest or experiences asymmetric shocks. This is also true if we consider two economies rather than a continuum of small open economies. The computation of the ergodic mean remains unaffected, as positive shocks in the rest of the union would cancel out with negative shocks for the home country, and the welfare results should be robust in that respect.

To compute welfare for the optimal transfer union we numerically search over the optimal transfers as a function of productivity shocks and then estimate the ergodic mean for welfare using second order perturbation methods.