Do Merger Efficiencies Always Mitigate Price Increases?[†]

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July 9, 2015

Abstract

In a Cournot model with differentiated products, we demonstrate that merger efficiencies in the form of lower marginal costs for the merging firms (the insiders) lead to higher post-merger prices under certain conditions. Specifically, when the degree of substitutability is low between the products offered by the two insiders but high between those by an insider and an outsider, increased merger efficiencies can exert upward rather than downward pressure on the prices of the merging firms. Our results suggest that in cases where firms engage in quantity competition, competition authorities should not presume that merger efficiencies will necessarily mitigate the anticompetitive effects of the merger. Prices can go up *because of* large efficiencies.

Key words: Merger efficiencies, Cournot model, Product differentiation

JEL Classification: L13, L40

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[†] We would like to thank Yongmin Chen, Gamal Atallah, Ping Lin, Chengzhong Qin, Aggey Semenov, Adam Wong, Wen Zhou and seminar participants at Shanghai University of Finance and Economics for their comments.

1. Introduction

Mergers can create efficiencies, arising from sources such as scale economy, rationalized production schedule between plants, and acquirement of complementary technologies. It is widely accepted by economists that lower marginal costs brought about by merger efficiencies will encourage firms to compete more aggressively, thus mitigating the loss of competition that may be caused by a merger. As Werden (1996, p.409) has noted, "If a merger caused a reduction in marginal cost for the merging firms, the cost reduction would offset the anticompetitive effect of the merger on prices. Indeed, if the merger reduced the marginal costs of the merging firms by a sufficient amount, it would cause all prices in the industry to fall."

Consistent with this conventional wisdom among economists, antitrust authorities typically associate the amount of price reduction with the magnitude of merger efficiencies. For example, the US Horizontal Merger Guidelines (2010, pp.30-31) states, "the Agencies consider whether cognizable efficiencies likely would be sufficient to reverse the merger's potential to harm customers in the relevant market, *e.g.*, by preventing price increases in that market." The EC Guidelines on the Assessment of Horizontal Mergers (2004, paragraph 79) indicates that "the relevant benchmark in assessing efficiency claims is that consumers will not be worse off as a result of the merger. For that purpose, efficiencies should be substantial…" In Canada, the *Competition Act* contains a provision that instructs the Competition Tribunal not to block a merger that "has brought about or is likely to bring about gains in efficiency that will be greater than, and will offset, the effects of any prevention or lessening of competition that will result or is likely to result from the merger..." Indeed, in discussions of merger enforcement policies and practices it is often held as self-evident that merger efficiencies mitigate anticompetitive effects and exert downward pricing pressure (see, *e.g.*, Fisher *et al*. 1989 and Salop 1987).

In this paper, we present a critical analysis of the conventional wisdom that merger efficiencies in the form of lower marginal costs will always counteract the price increases arising from the loss of competition. We do so by examining the effects of a merger in a model where firms produce differentiated products and compete in quantities. The merger generates efficiencies so that the marginal costs of the merging firms (the insiders) are lower after the merger. We show that an increase in merger efficiencies may either raise or reduce the post-merger prices of the insiders, depending on the degrees of substitutability among the products offered by the insiders and their rivals (the outsiders) as well as the number of competitors. Specifically, increased merger efficiencies exert upward – rather than downward – pressure on the prices if the degree of substitutability is low between the two insiders but high between the insiders and outsiders, and the number of competitors is not too small.

In the literature, theoretical analyses of merger efficiencies are typically conducted in the framework of Cournot oligopoly.[1] They include Farrell and Shapiro 1990, Levin 1990, Cheung 1992, Motta and Vasconcelos 2005, Banal-Estañol *et al*. 2008, Amir *et al*. 2009, Jovanovic and Wey 2012. All of them assume that firms produce a homogenous product, in which case a reduction in marginal costs after a merger always lead to a lower price. In contrast, our analysis shows that the effects of lower marginal costs depend critically on the degrees of product differentiation.

This paper is organized as follows. We derive our central result in a general model in section 2. We then elaborate on this result using a more specific model with linear demand functions in section 3. Conclusions are in section 4.

[1] One exception is Werden (1996), which studies merger efficiencies in a differentiated Bertrand model.

2. A General Model

Consider an industry where firms produce differentiated goods and compete in quantities. Initially, there are n (≥ 3) firms. Each firm produces one good at constant marginal cost, denoted by c. Then two of these firms, firm 1 and firm 2, decide to merge. The merger generates efficiency gains that reduce the marginal costs of the merging firms (the insiders) to $(1-e)c$, where $e \in (0,1)$ represents the magnitude of merger efficiencies. The marginal costs of the other firms (the outsiders) are not affected by the merger. The focus of our analysis will be on how the price effects of the merger are influenced by the size of e.[2]

On the demand side, we consider a situation where the demand functions are symmetric for the insiders and, respectively, for the outsiders. This enables us to analyze the equilibrium behavior of insiders and outsiders as two separate groups. Specifically, suppose that firm i $(= 1,2,\dots,n)$ faces the following (inverse) demand function: $p_i = P^i(\mathbf{q}_I, \mathbf{q}_o)$, where p_i is the price of product i, $\mathbf{q}_I = (q_1, q_2)$ the vector of quantities produced by the insiders (*i.e.*, firm 1 and firm 2), and $\mathbf{q}_o = (q_3, \dots, q_n)$ the vector of quantities produced by the outsiders (*i.e.*, firms 3 through n). We will use subscript j to denote the partial derivative of the demand function with respective to q_j. Thus, P^i_j $(\equiv \partial P^i / \partial q_j)$ indicates the responsiveness of price i to a small change in the quantity of good j. For $i \neq j$, P^i_j measures the degree of substitutability between good i and good j. Similarly, we use P^i_{jk} to represent $\partial^2 P^i / \partial q_k \partial q_j$.

Since goods are imperfect substitutes, we assume that $P^i_i < P^i_j < 0$ $(i, j = 1, \dots, n)$ in the relevant range of interest. Furthermore, we assume that the demand structure is symmetric and

[2] In this analysis, we do not consider merger efficiencies in the form of a reduction in fixed costs because its implication for post-merger prices is obvious. Provided that all outsiders remain active after the merger, the reduction in the insiders' fixed costs will have no impact on post-merger prices of all firms.

that firms within each group have the same degree of substitutability when they produce the same quantity. To be more precise, we assume that at $q_1 = q_2$ and $q_3 = q_4 = \cdots = q_n$,

- $P_j^i = P_i^j$ for all $i, j = 1, \ldots, n$;
- $P_j^i = P_k^i$ for all $i = 3, \ldots, n$ and all $j, k \neq i$;
- $P_j^1 = P_k^1$ and $P_j^2 = P_k^2$ for all $j, k = 3, \ldots, n$;
- $P_i^i = P_j^j$ for all $i, j = 3, \ldots, n$, and $P_1^1 = P_2^2$;
- $P_{ii}^i = P_{jj}^j$ and $P_{jj}^i = P_{ii}^j$ for all $i, j = 3, \ldots, n$, $P_{11}^1 = P_{22}^2$ and $P_{22}^1 = P_{11}^2$.

Note that the preceding assumption leaves open the possibility that $P_2^1 \neq P_j^1$ and $P_1^2 \neq P_j^2$ for all $j = 3, \ldots, n$. In other words, it permits the possibility that the degree of substitutability between the two insiders is different from the one between an insider and an outsider. This, in turn, enables us to investigate how the effects of the merger are influenced by the degree of substitutability between the two insiders.[3]

To simplify our analysis, we make an additional assumption that each inverse demand function is separable in its arguments. This assumption implies that $P_{jk}^i = 0$ whenever $j \neq k$. To ensure that the second-order conditions of a firm's profit-maximization problem are satisfied and that a firm's best-response function for quantity is decreasing in its rival's quantity of each product, we assume that $P_{jj}^i \leq 0$ for all $i, j = 1, \ldots, n$.

We will focus on interior equilibriums in which every firm produces a positive quantity both before and after the merger. Given the symmetry in the demand functions, it is convenient to discuss the equilibrium behavior of insiders and outsiders as two separate groups. We will

[3] In practice, one factor considered by competition authorities frequently in their assessment of the unilateral effects of a merger is whether the products of the merging parties are close substitutes relative to those offered by other competitors. See, for example, section 6.1 of the U.S. Horizontal Merger Guidelines (2010).

continue to use subscript I to denote the variables of an insider and subscript O those of an outsider.

Before the merger, each firm solves the following profit-maximization problem:

$$\max_{q_i} [P^i(\mathbf{q}_I, \mathbf{q}_o) - c]q_i. \qquad (1)$$

This yields the standard first-order condition:

$$P^i - c + q_i P_i^i = 0. \qquad (2)$$

Given the assumption of symmetric demand functions, both insiders produce the same quantity and all outsiders produce the same quantity in equilibrium. Using superscript C to indicate the pre-merger equilibrium, we write these quantities as $q_1 = q_2 = q_I^C$ and $q_3 = \cdots = q_n = q_o^C$.

After the merger, firms 1 and 2 coordinate their output decisions by solving

$$max_{q_1, q_2} [P^1(\mathbf{q}_I, \mathbf{q}_o) - (1-e)c]q_1 + [P^2(\mathbf{q}_I, \mathbf{q}_o) - (1-e)c]q_2. \qquad (3)$$

Note in (3) that the post-merger marginal costs of the insiders are $(1-e)c$. The first-order conditions can be written as:

$$P^i - (1-e)c + q_i P_i^i + q_j P_i^j = 0 \qquad (i, j = 1, 2, i \neq j). \qquad (4)$$

The optimization problem and the first-order condition of an outsider remain the same as (1) and (2). The post-merger equilibrium quantities are then determined by (2) and (4).

Using the demand symmetry, we proceed to characterize a symmetric equilibrium where both insiders produce the same quantity and all outsiders produce the same quantity. Let q_I^M and q_O^M denote the post-merger quantity of each insider and each outsider, respectively. Then (2) and (4) imply that q_I^M and q_O^M are determined by the following two equations:

$$P^1 - (1-e)c + q_I^M P_1^1 + q_I^M P_1^2 = 0; \qquad (5)$$

$$P^3 - c + q_o^M P_3^3 = 0. \qquad (6)$$

In (5) and (6), we use firm 1 as a representative of the insiders and firm 3 as a representative of the outsiders.

Before we proceed to analyze how merger efficiencies affect prices, we should comment on the profitability of the merger. After all, the discussion of merger efficiencies would be irrelevant if the merger is not profitable for the insiders (and hence they would not want to merge) in the first place. Indeed, it is well-known that a merger is usually not profitable in a homogeneous Cournot model where the merger generates no efficiency gains.[4] But the merger paradox does not arise in our model because of product differentiation and merger efficiencies. In the next section where we analyze a more specific model with linear demand functions, we will present the precise condition under which the merger is profitable.

The objective of this paper is to challenge the conventional wisdom that merger efficiencies in the form of lower marginal costs will always mitigate the price increases arising from the loss of competition. We achieve this by investigating how the post-merger quantities and prices are affected by e.

Conducting comparative statics on (5) and (6), we obtain:

$$\frac{\partial q_I^M}{\partial e} = \frac{-c[2P_3^3 + (n-3)P_4^3 + q_o^M P_{33}^3]}{[2P_1^1 + 2P_2^1 + q_I^M(P_{11}^1 + P_{11}^2)][2P_3^3 + (n-3)P_4^3 + q_o^M P_{33}^3] - 2(n-2)(P_3^1)^2}\,, \qquad (7)$$

$$\frac{\partial q_O^M}{\partial e} = \frac{2cP_1^3}{[2P_1^1 + 2P_2^1 + q_I^M(P_{11}^1 + P_{11}^2)][2P_3^3 + (n-3)P_4^3 + q_o^M P_{33}^3] - 2(n-2)(P_3^1)^2}\,. \qquad (8)$$

Using the properties of the demand functions, we find that the sign of (7) is positive while the sign of (8) is negative. In other words, larger merger efficiencies increase the quantity of each insider but reduce the quantity of each outsider.

[4] See, for example, Salant, *et al*. (1983) and Lommerud and Sørgard (1997).

The intuition behind the above results is straightforward. A lower marginal cost after the merger induces the insiders to expand their quantities. This causes each outsider to contract its output because quantities are strategic substitutes.

Using (7)-(8) and the demand functions, we can determine the impact of larger merger efficiencies on the price of an outsider:

$$\frac{\partial p_O^M}{\partial e} = \frac{-2cP_1^3(P_3^3 + q_o^M P_{33}^3)}{[2P_1^1 + 2P_2^1 + q_I^M(P_{11}^1 + P_{11}^2)][2P_3^3 + (n-3)P_4^3 + q_o^M P_{33}^3] - 2(n-2)(P_3^1)^2} < 0. \quad (9)$$

In other words, larger merger efficiencies lead to lower post-merger prices for the outsiders.

Our interest, however, is on the sign of $\partial p_I^M/\partial e$, *i.e.*, on what happens to the prices of the merging firms. This is because, in practice, a key question that an antitrust authority would ask is whether the merging firms will raise their prices after the merger. Again using (7)-(8) and the demand functions, we obtain the following result.

Proposition 1: A larger e leads to higher prices for the merging firms if and only if

$$\frac{P_2^1}{P_1^1} + 1 < \frac{2(n-2)P_1^3}{[2P_3^3 + (n-3)P_4^3 + q_o^M P_{33}^3]}\left[\frac{P_3^1}{P_1^1}\right]. \quad (10)$$

Proof: It can be shown that

$$\frac{\partial p_I^M}{\partial e} = \frac{c[2(n-2)P_3^1 P_1^3 - (P_1^1 + P_2^1)(2P_3^3 + (n-3)P_4^3 + q_o^M P_{33}^3)]}{[2P_1^1 + 2P_2^1 + q_I^M(P_{11}^1 + P_{11}^2)][2P_3^3 + (n-3)P_4^3 + q_o^M P_{33}^3] - 2(n-2)(P_3^1)^2}. \quad (11)$$

The denominator of (11) is positive. The sign of the numerator is positive if and only if (10) holds. ■

Proposition 1 suggests that larger efficiencies can indeed lead to higher post-merger prices for the insiders under some circumstances. A close examination of (10) indicates that this would arise if the degree of substitutability between the products of the two insiders, as measured by P_2^1/P_1^1, is not too large relative to that between the products of an insider and an outsider, P_3^1/P_1^1. Furthermore, it can be verified that the right-hand side of (10) increases with the number of firms (n). This implies that it is easier to satisfy condition (10) if the number of firms becomes larger.

A drawback of condition (10) is that it depends on endogenous variables. To gain a more complete understanding of this condition, we examine a model with linear demand functions in the next section.

3. Linear Demand Functions

Now suppose that the demand functions are linear:

$$p_1 = \alpha - q_1 - \gamma q_2 - \delta \sum_{j=3}^{n} q_j, \tag{12}$$

$$p_2 = \alpha - \gamma q_1 - q_2 - \delta \sum_{j=3}^{n} q_j, \tag{13}$$

$$p_i = \alpha - q_i - \delta \sum_{j \neq i} q_j \ \ (i = 3, 4, \dots, n), \tag{14}$$

where the values of γ and δ are strictly between 0 and 1. In terms of the general demand functions in section 2, we now have $P_i^i = -1$, $P_{ii}^i = 0$, $P_2^1 = P_1^2 = -\gamma$, and $P_j^i = -\delta$ for $i = 1, \dots, n$ and $j = 3, \dots, n$. Accordingly, γ measures the degree of substitutability between the two insiders (firms 1 and 2), while δ indicates the degree of substitutability between an insider and an outsider as well as that between any pair of outsiders. A larger γ (respectively, δ) means that the goods produced by the two insiders (respectively, by an insider and an outsider) are closer substitutes.

To ensure that the quantities produced by all firms are positive in the equilibriums before and after the merger, we assume that $\alpha > c$ and that $e < \bar{e}$, where $\bar{e} \equiv (\alpha - c)(1 + \gamma - \delta)/(\delta c)$. The second assumption is needed to ensure that each outsider produces a positive quantity after the merger. Intuitively, an outsider could be driven out of the market after the merger if the size of e is very large (and hence the marginal costs of the merging firms are very low). The precise role of this assumption can be seen below in the post-merger equilibrium quantities.

It can be verified that $\bar{e} > 1$ if $\alpha > c(1+\gamma)/(1+\gamma-\delta)$. Given that $e < 1$, the assumption $e < \bar{e}$ is not binding for this range of parameters.

3.1 Pre- and Post-Merger Equilibriums

Using the linear demand functions, we can obtain the closed-form solutions to the equilibrium quantities and prices before and after the merger. To simplify presentation, we define $A \equiv (n-3)\delta + 2$ and $B \equiv (n-2)\delta$. Since $\delta < 1$, it can be verified that $A > B$.

From the first-order condition (2), we derive the pre-merger equilibrium quantities of an insider and an outsider:[5]

$$q_I^C = \frac{(2-\delta)(\alpha-c)}{(2+\gamma)A-2\delta B}, \qquad q_O^C = \frac{(2+\gamma-2\delta)(\alpha-c)}{(2+\gamma)A-2\delta B}. \tag{15}$$

Using (15) and the demand functions, we find the equilibrium prices before the merger:

$$p_I^C = \frac{(\alpha-c)(2-\delta)}{(2+\gamma)A-2\delta B} + c, \;\; p_O^C = \frac{(\alpha-c)\,(2+\gamma-2\delta)}{(2+\gamma)A-2\delta B} + c. \tag{16}$$

Similarly, it can be shown that the post-merger equilibrium quantities and prices are:

$$q_I^M = \frac{(\alpha-c)(2-\delta)+ecA}{2[(1+\gamma)A-\delta B]}, \qquad q_O^M = \frac{(\alpha-c)(1+\gamma-\delta)-\delta ec}{(1+\gamma)A-\delta B}, \tag{17}$$

$$p_I^M = \frac{(\alpha-c)(1+\gamma)(2-\delta)-ec[(1+\gamma)A-2\delta B]}{2[(1+\gamma)A-\delta B]} + c, \qquad p_O^M = \frac{(\alpha-c)(1+\gamma-\delta)-\delta ec}{(1+\gamma)A-\delta B} + c. \tag{18}$$

The assumption $e < \bar{e}$ implies that the numerator of q_O^M in (17) is positive, thus ensuring that each outsider produces a positive quantity after the merger.

[5] The linear demand functions and constant marginal costs ensure that the second-order condition of each profit-maximization problem is satisfied.

3.2 Merger Profitability

Before we proceed to investigate how merger efficiencies affect prices, we take a closer look at the issue of merger profitability. As noted earlier, the presence of product differentiation and merger efficiencies mean that the merger paradox does not arise in this model. With the linear demand functions, we are able to determine precisely the set of parameters for which the merger is profitable.

Intuitively, we expect that the merger should be profitable if it generates sufficiently large efficiencies for the insiders. Indeed,

Proposition 2: The merger is profitable if $e > e_\pi$, where

$$e_\pi \equiv \frac{(\alpha - c)(2-\delta)}{cA}\left[\frac{(2+2\gamma)A - 2\delta B}{(2+\gamma)A - 2\delta B}\sqrt{\frac{1}{1+\gamma}} - 1\right]. \qquad (19)$$

Proof: The change in the profits of an insider is given by

$$\Delta\pi_I = \pi_I^M - \pi_I^C = \frac{(1+\gamma)[(\alpha - c)(2-\delta) + ecA]^2}{4[(1+\gamma)A - \delta B]^2} - \frac{(\alpha - c)^2(2-\delta)^2}{[(2+\gamma)A - 2\delta B]^2}. \qquad (20)$$

Rewriting (20) using the common denominator, we find that the corresponding numerator would be positive if and only if

$$(ecA)^2 L + ecA(\alpha - c)(2-\delta)2L + K > 0, \qquad (21)$$

where $L = (1+\gamma)[(2+\gamma)A - 2\delta B]^2$ and

$$K = (\alpha - c)^2(A-B)^2\gamma[(1+\gamma)\gamma A^2 - 4\delta(1+\gamma)AB + 4\delta^2 B^2]. \qquad (22)$$

The left-hand side of (21) is a quadratic function of e. Using the two roots of e, we find that (21) holds if $e > e_\pi$. ■

Note that the value of e_π may be negative, in which case the merger is profitable for any $e \geq 0$. This implies that the merger can be profitable even in the absence of efficiencies (*i.e.*, when $e = 0$).[6] The remaining analysis is conducted under the assumption that $e > \max\{0, e_\pi\}$, *i.e.*, the merger is profitable.

3.3 Merger Efficiencies

To investigate the effects of merger efficiencies on post-merger prices, we rewrite condition (10) in the general model using the linear demand functions and obtain

$$1 + \gamma < \frac{2(n-2)\delta^2}{2 + (n-3)\delta}. \qquad (23)$$

Since γ is positive, the right-hand side has to be greater than 1 in order to satisfy (23). Therefore,

Proposition 3: A larger e leads to higher prices for the merging firms if and only if

$$\gamma < \frac{2(n-2)\delta^2}{(n-3)\delta + 2} - 1. \qquad (24)$$

To satisfy (24), it is necessary that $\delta > 1/2$ and

$$n > \frac{2 + \delta(4\delta - 3)}{\delta(2\delta - 1)}. \qquad (25)$$

Proof: Condition (24) follows directly from (23). Since $\gamma > 0$, the right-hand side of (24) has to be positive. The latter is satisfied if and only if $\delta > 1/2$ and (25) holds. ■

Proposition 3 reinforces the finding in Proposition 1. It clearly shows that merger efficiencies exert upward pressure on the prices of the merging firms if the degree of substitutability is low between the two products of the insiders but high between those of an insider and an outsider, and the number of competitors is not too small.

[6] See Chen and Li (2015) for the specific conditions under which the merger is profitable in the absence of efficiencies.

This finding is in sharp contrast to the common belief among economists that lower marginal costs always lead to lower prices. To understand the intuition behind this surprising result, rewrite the insiders' demand functions in (12) as:

$$p_I = \alpha - (1+\gamma)q_I - \delta(n-2)q_O. \qquad (26)$$

From (26), we can see that merger efficiencies affect the insiders' prices through two channels. The first channel is through each insider's output q_I^M. In (7) we see that lower marginal costs induce the insiders to expand output after the merger. This is the *direct effect* of merger efficiencies on the post-merger prices of the insiders. The second channel is through every outsider's output q_O^M. From (8) we find that larger merger efficiencies cause each outsider to reduce its output. This is the *strategic effect* arising from the fact that outputs are strategic substitutes. As the insiders expand outputs in response to their lower marginal costs, the outsiders react by contracting their outputs. The latter tends to push up the insiders' prices.

As we can see from (26), the magnitude of the direct effect decreases with the degree of substitutability between the two insiders (γ), while the aggregate of the strategic effect for all outsiders increases with the number of competitors (n) and the degree of substitutability between the products of an insider and an outsider (δ). Accordingly, the strategic effect dominates the direct effect if n and δ are sufficiently large and γ is sufficiently small.

Three points about the conditions in Proposition 3 are worth noting. First, condition (24) implies that $\gamma < \delta$, *i.e.*, the degree of substitutability between the insiders is smaller than that between an insider and an outsider. In other words, larger merger efficiencies lead to higher prices only if the insiders' products are more distant substitutes for each other than for those of the outsiders.

Second, the positive relationship between post-merger prices and efficiencies does not arise if goods are homogeneous. To be more precise, the conditions in Proposition 3 cannot be satisfied by $\gamma = \delta = 1$. This is why merger efficiencies always lead to lower prices in the existing merger literature based on homogenous Cournot models (*e.g.*, Farrell and Shapiro 1990).

Third, condition (25) does not necessarily mean that this industry is unconcentrated. It can be shown that the expression on the right-hand side of (25) is decreasing in $\delta \in (1/2, 1]$ and is equal to 3 at $\delta = 1$. In other words, (25) could be satisfied in an industry with only four firms before the merger.

Proposition 3 points to an intriguing possibility that a merger could lead to lower prices when efficiencies are small, but higher prices when efficiencies are large. To examine this possibility, we focus on the case where the conditions in Proposition 3 hold.

Using (16) and (18), we write the difference between the post- and pre-merger prices of each insider as

$$\Delta p_I = p_I^M - p_I^C = \frac{[(1+\gamma)A - 2\delta B]\{\gamma(\alpha - c)(2-\delta) - ec[(2+\gamma)A - 2\delta B]\}}{2[(1+\gamma)A - \delta B][(2+\gamma)A - 2\delta B]}. \quad (27)$$

As will be shown below in the proof of Proposition 4, the sign of (27) depends on the sign of the second term in the numerator, namely,

$$\gamma(\alpha - c)(2-\delta) - ec[(2+\gamma)A - 2\delta B]. \quad (28)$$

From (28) we derive another critical value of e:

$$e_p \equiv \frac{\gamma(\alpha - c)(2-\delta)}{c[(2+\gamma)A - 2\delta B]}. \quad (29)$$

Comparing it with the other two critical values of e, we find the following relationship.

Lemma 1: $e_\pi < e_p < \bar{e}$**.**

Proof: It is clear that $e_p > 0$. Then $e_p > e_\pi$ if $e_\pi \leq 0$. In the case where $e_\pi > 0$, straightforward manipulations of (19) and (29) yield that $e_p > e_\pi$. To prove that $e_p < \bar{e}$, we need to show that

$$\frac{\gamma(\alpha - c)(2-\delta)}{c[(2+\gamma)A - 2\delta B]} < \frac{(\alpha - c)(1+\gamma-\delta)}{\delta c}, \qquad (30)$$

which is equivalent to $\delta\gamma(2-\delta) < (1+\gamma-\delta)[(2+\gamma)A - 2\delta B]$. Since $\delta\gamma(A-B) < (1+\gamma-\delta)(2\delta A - 2\delta B) < (1+\gamma-\delta)[(2+\gamma)A - 2\delta B]$ and $A - B = 2 - \delta$, it follows that $e_p < \bar{e}$. ■

In light of (27), Lemma 1 implies that e_p divides the interval $(e_\pi, \bar{e})$ into two segments. The sign of (27) is positive for e in one segment and negative in the other segment. This suggests that a profitable merger can either raise or reduce the prices of the insiders.

Proposition 4: Suppose $\delta > 1/2$ and (24)-(25) are satisfied. If

(a) $\alpha < c(2-\delta+A)/(2-\delta)$, or

(b) $\alpha > c(2-\delta+A)/(2-\delta)$ and $\gamma < 2c(A-\delta B)/[(\alpha-c)(2-\delta) - cA]$,

then the merger reduces the merging firms' prices, *i.e.*, $p_I^M < p_I^C$ for $e \in (e_\pi, e_p)$, but it raises their prices, *i.e.*, $p_I^M > p_I^C$, for $e \in (e_p, \min\{\bar{e}, 1\})$.

Proof: Observe that the denominator of (27) is positive. In the numerator of (27), $(1+\gamma)A - 2\delta B < 0$ because of (24). Then Δp_I has the opposite sign of (28). The latter depends on the value of e relative to e_p. From (29), we find that $e_p < 1$ if the condition(s) in either (a) or (b) is satisfied. Then $\Delta p_I < 0$ if $e < e_p$ and $\Delta p_I > 0$ if $e > e_p$.

Note that $e_\pi > 0$ in this case. This can be proved by using (19) to show that $e_\pi > 0$ if $\gamma < \left[4\delta B - A + \sqrt{A^2 + 8\delta AB}\right]/2A$. The latter is true by $\left[4\delta B - A + \sqrt{A^2 + 8\delta AB}\right]/2A > (2\delta B - A)/A$ and (24). ■

Two implications of Proposition 4 are worth noting. First, the merger can lead to lower prices. This is not surprising given that the merger reduces the marginal costs of the insiders. Second and more interestingly, the post-merger prices are lower only when the efficiencies are small. Large efficiencies actually lead to high prices under some circumstances.

Clearly, Proposition 4 does not cover all possible scenarios in this model. In particular, it does not include those parameter values for the "normal" scenarios where efficiencies do mitigate the price increases caused by the merger. Given the focus of this paper, however, we relegate the discussions of these scenarios to the appendix.

The policy implication of Propositions 3 and 4 is that in cases where firms engage in quantity competition, competition authorities should not presume that merger efficiencies will necessarily counteract the anticompetitive effects of the merger. The merger can lead to higher prices *because of*, rather than despite of, large efficiencies.

3.4 Numerical Examples

Finally, we present a pair of numerical examples to illustrate our results. These examples will also show that the positive relationship between efficiencies and post-merger prices can arise in situations where the merger causes significant price increases. This is important for the policy implications of our findings. Competition authorities are usually concerned about mergers that could lead to significant increases in prices. If our results should arise only in situations where the merger leads to small price increases, they would be mere theoretical possibilities that are of little concern to competition authorities. Therefore, these numerical examples serve to demonstrate that our results are of practical relevance to merger enforcement policy.

With this in mind, we choose the parameter values so that the merger leads to price increases for the insiders in excess of 10 percent.[7] Specifically, we set $n = 9$, $\gamma = 0.1$, $\delta = 0.9$, $\alpha = 3$, and $c = 1$. We choose two values for the size of merger efficiencies: $e = 0.30$ and $e = 0.31$.

The pre- and post-merger equilibrium prices, quantities and profits of insiders and outsiders are presented and compared in Tables 1 and 2. In particular, the last three columns show the magnitude of price change (Δp), the percentage change in price ($\Delta p/p^C$), and the change in profits ($\Delta\pi$).

Table 1: Effects of the Merger ($e = 0.30$)

	p^C	p^M	q^C	q^M	π^C	π^M	Δp	$\Delta p/p^C$	$\Delta\pi$
Insiders	1.524	1.684	0.524	0.895	0.274	0.612	0.160	10.5%	0.338
Outsiders	1.143	1.053	0.143	0.053	0.020	0.003	-0.090	-7.9%	-0.018

Table 2: Effects of the Merger ($e = 0.31$)

	p^C	p^M	q^C	q^M	π^C	π^M	Δp	$\Delta p/p^C$	$\Delta\pi$
Insiders	1.524	1.691	0.524	0.910	0.274	0.628	0.167	11.0%	0.354
Outsiders	1.143	1.049	0.143	0.049	0.020	0.002	-0.094	-8.2%	-0.018

From Tables 1 and 2, we see that in both instances the merger is profitable for the insiders. Moreover, the merger enables the insiders to raise their prices substantially, by 10.5 percent in the case of $e = 0.30$ and 11.0 percent in the case of $e = 0.31$.

[7] To put this in perspective, note that 5 percent is commonly used in the hypothetical monopolist test as the threshold for a significant price increase.

This pair of numerical examples confirms the central result of this paper. Starting from the same pre-merger price of 1.524, the insiders raise their prices to 1.684 if the merger efficiencies are $e = 0.30$. If the merger efficiencies are increased to $e = 0.31$, the insiders hike their prices even further to 1.691. Here, larger efficiencies exacerbate, rather than mitigate, the price increases by the insiders.

4. Conclusions

To answer the question posed in the title of this paper, merger efficiencies do not necessarily mitigate the price increases arising from the loss of competition. Our analysis demonstrates that when firms compete in quantity, lower marginal costs after a merger can exert upward rather than downward pressure on the prices of the merging firms. A policy implication of our results is that in cases where firms engage in quantity competition, a competition authority can no longer presume that merger efficiencies will necessarily offset the anticompetitive effects of the merger. The merger can lead to higher prices *because of*, rather than despite of, large efficiencies.

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Appendix

Additional Analysis of the Price Effects of Merger

From the conditions in Proposition 4, we can see that there are many possible configurations for the ranges of α, γ, δ and n. To reduce the number of cases we have to discuss and hence to simplify the presentation, we will focus on the cases where $\delta > 1/2$ and n satisfies (25). These restrictions on δ and n ensure that $2\delta B - A > 0$.

To simplify notations, define

$$\alpha_1 \equiv \frac{c(2-\delta+A)}{2-\delta}.$$

Moreover, define α_2 as the solution to

$$\frac{2c(A-\delta B)}{(\alpha_2 - c)(2-\delta) - cA} = \frac{2\delta B - A}{A}.$$

That is,

$$\alpha_2 \equiv \frac{c[2(A-\delta B)A + (2\delta B - A)(2-\delta+A)]}{(2-\delta)(2\delta B - A)}.$$

It can be verified that $\alpha_1 < \alpha_2$. Moreover,

$$\frac{2c(A-\delta B)}{(\alpha - c)(2-\delta) - cA} < \frac{2\delta B - A}{A}$$

if and only if $\alpha > \alpha_2$.

In terms of these notations, the two cases considered in Proposition 4 are

Case (a) $\alpha < \alpha_1$ and $\gamma < (2\delta B - A)/A$; and

Case (b) $\alpha > \alpha_1$ and $\gamma < \min\{(2\delta B - A)/A, 2c(A-\delta B)/[(\alpha - c)(2-\delta) - cA]\}$.

Below we discuss four remaining configurations of parameter values.

Case (c) $\alpha < \alpha_1$ and $\gamma > (2\delta B - A)/A$.

In this case, $e_p < 1$, $(1+\gamma)A - 2\delta B > 0$, and hence the sign of (27) is the same as that of (28). From (28), we conclude that $p_I^M > p_I^C$ for $e < e_p$, and $p_I^M < p_I^C$ for $e > e_p$. That is, the merger raises the prices of the insiders if the efficiencies are small, but reduces the prices of the insiders if the efficiencies are large.

Case (d) $\alpha > \alpha_1$ and $\gamma >$ max $\{(2\delta B - A)/A, 2c(A - \delta B)/[(\alpha - c)(2-\delta) - cA]\}$.

In this case, $e_p > 1$, $(1+\gamma)A - 2\delta B > 0$, and hence the sign of (27) is the same as that of (28). Since the sign of (28) is positive for all $e < 1 < e_p$, we conclude that $p_I^M > p_I^C$ for any $e < 1$; that is, the merger raises the prices of the insiders for any size of efficiencies. Here, larger efficiencies mitigate the price increases.

Case (e) $\alpha \in (\alpha_1, \alpha_2)$ and $(2\delta B - A)/A < \gamma < 2c(A - \delta B)/[(\alpha - c)(2-\delta) - cA]$.

In this case, $e_p < 1$, $(1+\gamma)A - 2\delta B > 0$, and hence the sign of (27) is the same as that of (28). The results here are qualitatively the same as in Case (c), *i.e.*, $p_I^M > p_I^C$ for $e < e_p$, and $p_I^M < p_I^C$ for $e > e_p$.

Case (f) $\alpha > \alpha_2$ and $2c(A - \delta B)/[(\alpha - c)(2-\delta) - cA] < \gamma < (2\delta B - A)/A$.

In this case, $e_p > 1$, $(1+\gamma)A - 2\delta B < 0$, and hence the sign of (27) is the opposite to that of (28). Since (28) is positive for all $e < 1 < e_p$, we conclude that $p_I^M < p_I^C$ for any $e \in (e_\pi, 1)$; that is, the merger reduces the prices of the insiders for any size of efficiencies that make the merger profitable. However, larger efficiencies counteract the price reduction.